Doctoral School of the Faculty of Applied Sciences

Session Program
MATHEMATICS

BNs 05 - 25.05.2017

OPENING SESSION **9:00-9:20**

Welcome address *Prof. Andrei HALANAY, Assistant Director of Doctoral School*

Opening address *Eng. Drd. Daniela ENCIU, Doctoral School of the Faculty of Applied Sciences*

SESSION ONE **9:20-10:20**

Chair, *Ana-Maria BORDEI, Doctoral School of the Faculty of Applied Sciences*

1. Implications of random processes theory by ms excel and origin in the study of linear dynamic systems
Bogdan Cioruța, Nicolae Pop, Tudor Sireteanu

2. A functional equation of butler-rassias type and its Hyers-ulam stability
Mihai Monea

3. Hydraulic servomechanism models with delay
Daniela Enciu, Andrei Halanay, Ioan Ursu

PAUSE **10:20-10:40**

SESSION TWO **10:40-11:20**

Chair, *Daniela ENCIU, Doctoral School of the Faculty of Applied Sciences*

4. Stochastic connectivity on almost-Riemannian structures induced by symmetric polynomials
Teodor Ţurcanu, Constantin Udrişte

5. Stability analysis of some equilibrium points in a complex model for cells evolution in leukemia
Ana-Maria Bordei, Irina Badralexi, Andrei Halanay

CLOSING SESSION **10:20-10:40**

Chair, *Ana-Maria BORDEI, Doctoral School of the Faculty of Applied Sciences*

IMPLICATIONS OF RANDOM PROCESSES THEORY BY MS EXCEL AND ORIGIN IN THE STUDY OF LINEAR DYNAMIC SYSTEMS

Bogdan CIORUȚA[1,3], Nicolae POP[2], Tudor SIRETEANU[2,3]
[1]*North University Centre at Baia Mare - Technical University of Cluj-Napoca, Faculty of Engineering*
Victor Babeş str., no. 62A, 430083, Baia Mare
bciorutza@yahoo.com
[2]*Institute of Solid Mechanics of the Romanian Academy*
Constantin Mille str., no. 15, sector 1, Bucharest
[3]*University Politehnica of Bucharest - Faculty of Applied Sciences*
Splaiul Independenţei, no. 313, RO-060042, sector 6, Bucharest

Abstract (ro):

Since ancient times, the phenomena of nature and society were present in people's daily scientific activities, and as society evolved, their importance in the complex system of human civilization has steadily increased. The study of nature and society was led scientists to create theories and mathematical models that include the main characteristics of abstract forms. For the phenomena of nature and society, which are originally evolutionary phenomena (dynamic), the best model was found to be the result of specific observation and measurement-based methods, concepts at the core of natural and technical sciences.

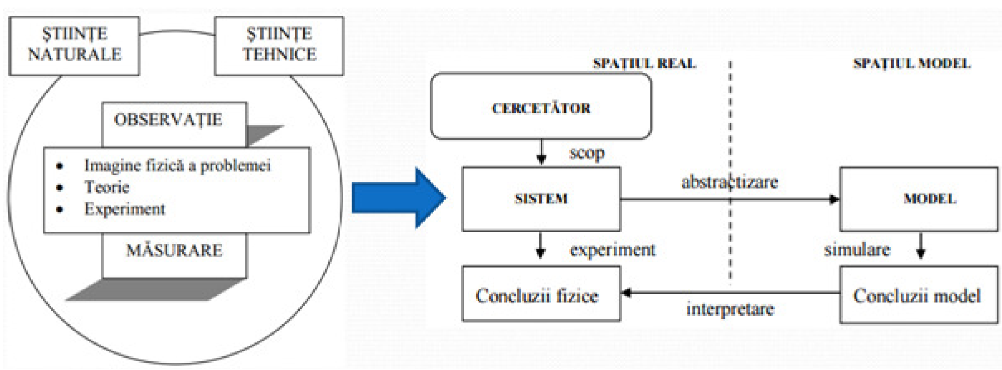


Figure 1. Position of the measurement and evaluation concepts in relation to natural and technical sciences

The main purpose of dynamical systems theory, which we have raised, is to understand the long-term behavior of states of a system, most often deterministic. For this study, we follow, from concepts specific systems theory, achieving a ranking non-exhaustive patterns or systems, depending on the linearity, of their number of input-output variables, of behavior over time or taking into account other aspects . Often, such systems involve many variables and are nonlinear. Therefore, the study of the behavior of dynamic systems require graphics (modeling and simulation) particularly complex that can be easily done with computers from nowadays, some of them will be presented in the paper.

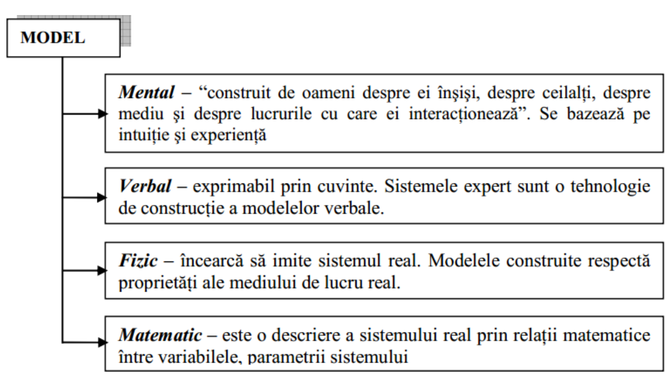
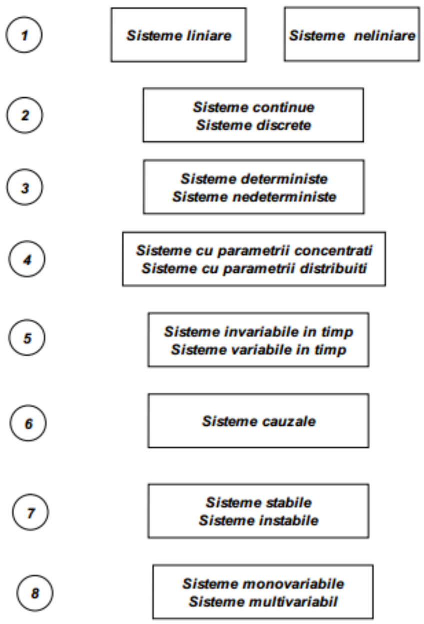


Figure 2. Various models and their representation



Tipul de model	Caracterizare
Modele utilizate în scopul cercetării	utilizate ca un instrument de cercetare
Modele utilizate în management	utilizate ca un instrument de management
Modele deterministe	valorile prezise au valoare generată exact depind numai de datele de intrare
Modele stocastice	valorile prezise depind de distribuții probabilistice
Modele reducționiste	includ pe cât posibil toate detaliile relevante
Modele holiste	utilizează principii generale
Modele statice	variabilele care definesc sistemul nu sunt dependente de timp
Modele dinamice	variabilele care definesc sistemul sunt funcție de timp sau spațiu
Modele distributive	parametrii sunt considerați funcții de timp și spațiu
Modele agregative (engl."lumped")	definite la scară spațială mare, exprimate prin valori medii ale variabilelor de stare și proces
Modele lineare	considerate ecuații de gradul I
Modele nelineare	una/mai multe ecuații nu sunt de gradul I, ecuații de grad superior sau funcții neliniare
Modele de cauzalitate	intrările, variabilele de stare, ieșirile sunt interrelaționate prin relații de cauzalitate
Modele necauzale (engl."black-box")	intrările în sistem sunt comparate cu ieșirile fără analiza mecanismului intern cauzal
Modele autonome/independente	derivatele nu sunt în mod explicit dependente de variabila independentă (timpul)
Modele dependente	derivatele sunt în mod explicit dependente de variabila independentă (timpul)

Figure 3. Types of physical (real) systems and associated models

The fundamental feature of dynamic systems involving the movement and the objective existence in time and space of physical systems they set up purely mathematical, as mature fruits of researchers thought.

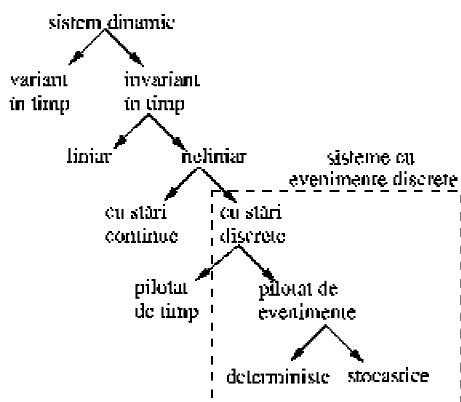


Figure 4. Overall classification of dynamic mechanical systems

The study of natural systems and processes is based on the causality principle; physical systems shows mechanical, thermal, electrical and magnetic properties, which can be analyzed in successive stages.

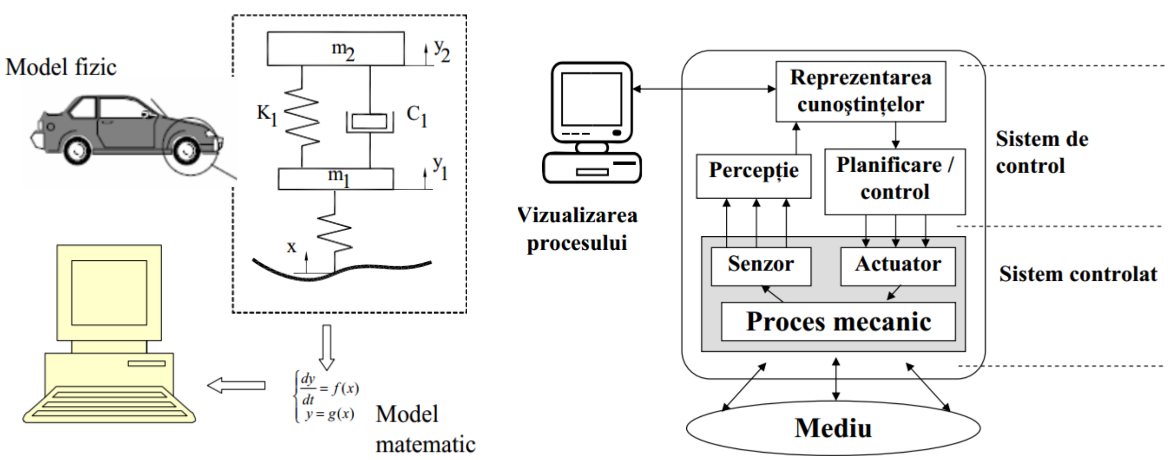
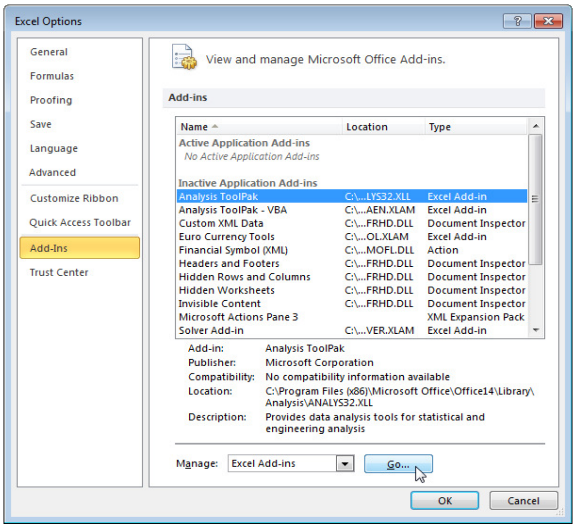
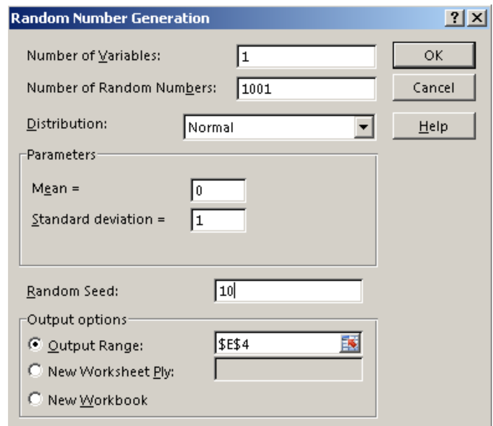


Figure 5. Specific steps in the systemic analysis methodology and behavior control

Considering the stated, the present study shows, in addition to the above mentions, concepts which have a significant role in the study of linear dynamic systems, from the random processes theory perspective, among which we considered the distribution function, density function of probability, statistical moments of a random process, and the correlation (cross-correlation) of a random signal (process). A special place is allocated, towards the end, to the methodology that generates a and analyzes the random sets of data (variables) through the software readily available, namely, MS Excel, respectively Origin.





Parameters		random seed	10	20	30	40	50	60	70	80	90	100
		time (t)	$x_1(t)$	$x_2(t)$	$x_3(t)$	$x_4(t)$	$x_5(t)$	$x_6(t)$	$x_7(t)$	$x_8(t)$	$x_9(t)$	$x_{10}(t)$
mean	0	0	-2.85283	-2.73241	-2.63957	-2.56507	-2.50431	-2.45007	-2.40219	-2.3605	-2.32141	-2.28558
st. deviation	1	0.01	0.03944	0.82704	-1.43235	-0.36868	0.3487	1.38192	-0.85361	-0.05813	0.69663	-1.78525
		0.02	-1.28237	0.12961	-2.71986	-0.11304	1.32073	-0.36271	0.87843	-0.63766	0.5638	-0.97308
		0.03	-0.20722	0.71103	-1.25366	-0.12937	0.81365	-1.10033	-0.05223	0.92577	-0.96915	0.02452
		0.04	-0.20698	0.16557	0.5637	1.08615	-2.36425	-1.00781	-0.51137	-0.11981	0.25349	0.66766
		0.05	0.14683	0.69575	1.69621	-1.02202	-0.38067	0.1259	0.66958	1.61436	-1.05762	-0.40305
		0.06	-0.82338	-1.01533	-1.2545	-1.60127	-2.6108	1.68884	1.30605	1.05401	0.8557	0.68633
		0.07	-1.32901	-0.84658	-0.50928	-0.22256	0.04672	0.31941	0.61874	0.98879	1.60099	-1.6263
		0.08	-0.36418	-0.26163	-0.16177	-0.0635	0.03416	0.13207	0.23135	0.33297	0.43815	0.54844
		0.09	-1.57218	-0.90876	-0.50867	-0.1783	0.13331	0.45871	0.84352	1.43471	-1.6642	-0.94712
		0.1	-1.11005	-1.35098	-1.71454	2.89561	1.67437	1.32753	1.09252	0.90611	0.74685	0.60453
		0.11	1.13127	1.19757	1.26976	1.34908	1.43814	1.54009	1.66146	1.81343	2.02519	2.40911
		0.12	0.81035	0.11904	-0.51399	-1.55606	0.9007	0.18242	-0.44372	-1.37322	0.99908	0.24654
		0.13	-1.57086	-1.23918	-1.00603	-0.81781	-0.65491	-0.5078	-0.3709	-0.24071	-0.1145	0.00991
		0.14	-0.25523	-0.94724	1.59197	0.57342	-0.02682	-0.63775	-1.82101	0.86634	0.20027	-0.38034
		0.15	0.76231	-1.48927	-0.36018	0.38676	1.56929	-0.72947	0.06028	0.89726	-1.24614	-0.25974
		0.16	-0.1075	0.36214	0.93687	-2.33522	-0.86334	-0.31025	0.15704	0.66384	1.48189	-1.20039
		0.17	-0.18888	-1.66451	0.44262	-0.54205	1.38411	0.0998	-0.98345	0.79138	-0.23127	-1.85969
		0.18	0.38577	-0.81046	0.73077	-0.4504	1.20008	-0.14181	-2.83537	0.15348	-1.17667	0.46322
		0.19	0.32433	0.86445	2.18359	-0.97566	-0.40214	0.0576	0.53056	1.18142	-1.5512	-0.70759
		0.2	-1.15583	0.27313	-1.33254	0.18935	-1.56485	0.10688	-1.93858	0.02513	2.49907	-0.05645
		0.21	0.51766	-0.1966	-1.05174	1.13054	0.24118	-0.46859	-1.70498	0.73397	-0.01779	-0.78127
		0.22	-0.69663	0.93544	-0.23434	2.30913	0.18071	-1.02009	0.63111	-0.47286	1.28359	-0.04389
		0.23	1.48556	-1.46133	-0.79725	-0.37631	-0.01496	0.34439	0.7568	1.37934	-1.58253	-0.85075
		0.24	-2.05903	-0.05959	1.49649	-0.28148	1.01803	-0.51828	0.7019	-0.78897	0.44422	-1.13578
		0.25	-0.21402	0.80685	-0.97418	0.10011	1.37068	-0.55449	0.42472	-1.75842	-0.21668	0.80336

	$x_1(t)$	$x_2(t)$	$x_3(t)$	$x_4(t)$	$x_5(t)$	$x_6(t)$	$x_7(t)$	$x_8(t)$	$x_9(t)$	$x_{10}(t)$
	-2.8528	*-2.7324*	*-2.6396*	*-2.5651*	*-2.5043*	*-2.4501*	*-2.4022*	*-2.3605*	*-2.3214*	*-2.2856*
Mean	0.04678	-0.0423	-0.0672	0.02098	-0.0116	0.01632	-0.0206	0.04155	0.02523	-0.0601
Standard Error	0.03234	0.03215	0.03132	0.03112	0.03101	0.03108	0.03087	0.03139	0.03179	0.03067
Median	0.06334	-0.0583	-0.0833	0.00298	-0.0158	0.05269	-0.0142	0.02961	0.06952	-0.0726
Mode	0.73808	0.69575	-2.7199	-0.9981	1.67437	-0.6377	0.82145	-0.1905	0.54214	-0.3902
Standard Deviation	1.02277	1.01671	0.99056	0.98421	0.98069	0.9827	0.97611	0.99268	1.00525	0.96997
Sample Variance	1.04606	1.03371	0.98122	0.96866	0.96175	0.96569	0.9528	0.98542	1.01052	0.94083
Kurtosis	-0.0108	0.05151	-0.0444	-0.2903	0.20504	-0.0606	-0.2482	-0.1489	-0.3353	0.00053
Skewness	-0.072	-0.0523	-0.0098	0.10834	-0.0759	-0.0267	-0.0267	-0.0681	-0.1133	0.17731
Range	6.32601	6.833	6.33096	5.39076	6.89994	6.43078	5.86595	6.83794	5.74131	6.3538
Minimum	-3.046	-3.4558	-3.6682	-2.3821	-3.5227	-3.4558	-2.9948	-3.3152	-3.1067	-2.8311
Maximum	3.28	3.37721	2.66271	3.00861	3.37721	2.97499	2.87117	3.52273	2.63462	3.52273
Sum	46.7815	-42.255	-67.178	20.9776	-11.619	16.3186	-20.591	41.5499	25.2257	-60.053
Count	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

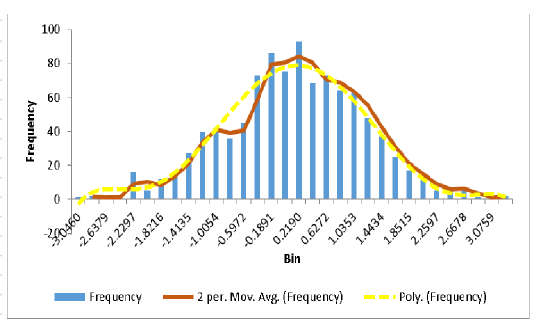


Figure 6. The normal distribution generation, in relation to given average and standard deviation data, of some random data sets, using Data Analysis package –Random Number Generation module

Keywords: *dinamic systems, random variables, Analysis ToolPak, Origin.*

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A FUNCTIONAL EQUATION OF BUTLER-RASSIAS TYPE AND ITS HYERS-ULAM STABILITY

MONEA MIHAI

The aim of this paper is to solve a trigonometric functional equation of Butler-Rassias type and its pexiderized version. Some Hyers-Ulam stability results are also stated.

We will present some results concerning the solution as well as the Hyers-Ulam stability of the functional equation

$$f(x+y) = af(x)\cos y + bf(y)\cos x,$$

where $f:\mathbb{R}\to\mathbb{R}$ and a, b are real parameters. Such type of functional equations that define well – known functions, or classical trigonometric formulas were studied by many authors. Our functional equation in case $a = b = 1$ is the equation defining the sine function, modulo a real constant c, as we can see in the sequel. In this case, the following trigonometric formula is obtained:

$$\sin(x+y) = \sin x\cos y + \cos x\sin y.$$

Following the published works of a number of mathematicians such equations are known as Butler – Rassias equations.

In the final section we study the following pexiderized equation:

$$f(x+y) = ag(x)\cos y + bh(y)\cos x,$$

where $f, g, h:\mathbb{R}\to\mathbb{R}$ and a, b are any nonzero real numbers.

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University Politehnica of Bucharest
E-mail address: mihaimonea@yahoo.com

Hydraulic servomechanism models with delay

Daniela Enciu[1,2], Andrei Halanay[2], Ioan Ursu[2]

[1]National Institute for Aerospace Research "Elie Carafoli" – INCAS,
Bd. Iuliu Maniu 220, Bucharest 061126, Romania
[2]University Politehnica of Bucharest, Mathematics Department 2,
Splaiul Independentei 313, Bucharest 060042, Romania

The problem of modelling hydraulic servomechanisms as systems with delay is relative new, although the delay is objective and real in every system. Probably this situation is given by the difficulties that can be encountered in modelling a state delay as derived from specific nonlinearities like dry friction in hydraulic cylinder. This direction may seem less natural than the introduction of the delay due to a physical law, such as the speed of propagation of a wave, for example, in the case of other systems.

The objective of this paper is to set both of a state of the art in the domain of hydraulic servomechanisms and introducing state delays as a technique for simplifying the model in the view of a qualitative study of the effects of the delay. The hydraulic servomechanism (mechanical or electrical) is an automatic system. Therefore two possible "locations" for the delay are taken into account: on state and on control.

In the specialty literature, in recent years, interest has emerged for assimilating the LuGre friction model with a delayed state model. An older approach is due to V. A. Hohlov where the nonlinear term from the mathematical model of the servomechanism flow equation is replaced with a state-delay. The delay on the control is justified and unavoidable in every automatic system. Often, the delay coming from the sensors is studied. The signal sampling, as usual procedure for elaborating and implementing control laws, also introduces delays in the systems as well. The compensation techniques of the delay effects are multiple and are studied in both time and frequency domain. One technique is the Smith predictor.

The paper ends with a few conclusions regarding the objectives of future studies in the mathematical modelling of the hydraulic systems with state and/or control delays.

Stochastic connectivity on almost-Riemannian structures induced by symmetric polynomials

Teodor Ţurcanu, Constantin Udrişte

1 Short presentation

In this paper we introduce an almost-Riemannian structure which is induced by the exact differential $1-$forms $\omega_1 = dx + dy,\ \ \omega_2 = ydx + xdy$. These are obtained by taking the differential of the elementary symmetric polynomials, defined on the real plane $\mathbb{R}^2$, endowed with the coordinates (x, y). Clearly, the given $1-$forms are functionally dependent on the set $\{x = y\}$ and functionally independent elsewhere. The kernels of the $1-$forms ω_1 and ω_2 are generated by the vector fields

$$X_1 = \partial_x - \partial_y \text{ and } X_2 = x\partial_x - y\partial_y,$$

respectively, which span the distribution $\mathcal{D} = span\{X_1, X_2\}$, inducing an almost-Riemannian structure on $\mathbb{R}^2$. The *singular locus*, i.e., the set of points at which the vector fields X_1 and X_2 lose their linear independence, is the set $S = \{(x, y) \in \mathbb{R}^2 |\ x = y\}$.

The sub-Riemannian metric g, which turns the pair $\{X_1, X_2\}$ into an orthonormal basis at each point $p \in \mathbb{R}^2 \backslash S$, is given by

$$g = (g_{ij}) = \frac{1}{(x-y)^2} \begin{pmatrix} 1+y^2 & 1+xy \\ 1+xy & 1+x^2 \end{pmatrix}, \tag{1}$$

and clearly is singular on S.

The almost-Riemannian structure defined above resembles, in some aspects, the famous (in the context of sub-Riemannian geometry) Grushin plane (with the structure induced by the vector fields $\partial_x, x\partial_y$) studied by many authors (see for instance [1, 3, 4, 8, 9]) from various viewpoints.

Recall that, given a smooth manifold M, a sub-Riemannian structure on M is specified by a given distribution $\mathcal{D}$, i.e., a sub-bundle $\mathcal{D} \subseteq TM$, together with a metric g defined on $\mathcal{D} \times \mathcal{D}$. Most often, the distribution is specified by a family of vector fields and is non-integrable.

The natural curves on sub-Riemannian manifolds are *horizontal (admissible) curves*, which are tangent to horizontal vectors. Thus, a classical problem, in the context of sub-Riemannian geometry, is to join two arbitrary points by admissible curves. A sufficient condition for this to be possible is that the vector fields, together with their iterated Lie brackets span the entire tangent space at each point $p \in M$. This fact is established by the famous Chow-Rashevskii Theorem [10, 14].

It is easily verified that, in this case, the bracket generating condition is satisfied:

$$[X_1, X_2] = \partial_x + \partial_y,$$

i.e., the Carnot-Carathéodory distance d_C between any two points is finite, moreover, the topology induced by the metric d_C is equivalent to the Euclidean topology (Corollary 2.6 [3]).

In this paper we are interested in stochastic analogues of connectivity problems on almost-Riemannian manifolds. To our knowledge, this problem was raised and motivated, for an arbitrary sub-Riemannian manifold, in [6, 7]. The problem has been solved for the Grushin plane and its generalizations in [6, 7, 16, 17, 18]. The main result of the paper is Theorem 1.2, which proves the stochastic connectivity property with respect to the almost-Riemannian structure defined above.

It is worth mentioning that when passing to a stochastic setting some important adjustments need to be made. Firstly, the admissible curves are replaced by *admissible stochastic processes* (defined below). Secondly, we have to replace the deterministic boundary conditions as well, since probability of an admissible stochastic process, starting at a state P, to reach a fixed state Q, is zero. Thus, we ask for a time at which the state of the process is in an arbitrarily small neighborhood of the state Q, with the probability close enough to one.

Any admissible curve $c : [0, T] \to \mathbb{R}^2$, between two fixed points P and Q is described by the boundary value problem

$$\begin{cases} dx(t) = [u_1(t) + u_2(t)x(t)]\, dt \\ dy(t) = -\, [u_1(t) + u_2(t)y(t)]\, dt \\ (x(0), y(0)) = (x_P, y_P)\,, \quad (x(T), y(T)) = (x_Q, y_Q)\,, \end{cases} \tag{2}$$

for some control functions $u_1, u_2 \in L^1([0, T], \mathbb{R})$. Using a pair of independent Wiener processes $\left(W_t^1, W_t^2\right)$, together with a pair of nonnegative constants (σ_1, σ_2), the ODE system (2) is stochastically perturbed, yielding the SDE system

$$\begin{cases} dx(t) = [u_1(t) + u_2(t)x(t)]\, dt + \sigma_1 dW_t^1 \\ dy(t) = -\, [u_1(t) + u_2(t)y(t)]\, dt + \sigma_2 dW_t^2. \end{cases} \tag{3}$$

Here the controls $u_i(s) = u_i(s, \omega)$, $i = 1, 2$ are stochastic processes measurable with respect to the $\sigma-$algebra generated by $\{W_{s\wedge t}, t \geq 0\}$, taking values in a Borel set at any instant. The controls which do not depend on ω are called *deterministic* or *open loop controls*. Controls of the form $u(s, \omega) = u_0\,(t, c_t\,(\omega))$, for some function u_0, are called *Markov controls*. Denote the set of deterministic controls by $\mathcal{U}_1$ and, respectively, the set of Markov controls by $\mathcal{U}_2$.

Definition 1.1. A stochastic process $c_t = (x(t), y(t))$ solving the SDE system (3) is called *admissible stochastic process*.

Theorem 1.2. *Let $P(x_P, y_P)$ and $Q(x_Q, y_Q)$ be two given points on $\mathbb{R}^2$ and denote by $D_C(Q, r)$ the Carnot-Carathéodory disk of radius r centered at Q. Then, for any $\varepsilon \in (0, 1)$ and $r > 0$, there exist an admissible stochastic process $c_t = (x(t), y(t))$, and a striking time $T < \infty$, such that*

$$\mathbb{P}\left[c_T \in D_C(Q, r)\right] \geq 1 - \varepsilon, \tag{4}$$

and $x(0) = x_P$, $y(0) = y_P$, $\mathbb{E}\left[y(T)\right] = y_Q$, $\mathbb{E}\left[x(T)\right] = x_Q$.

2 A three dimensional generalization

In the present section we discuss a generalization of previously obtained result in a three-dimensional setting, i.e., on $\mathbb{R}^3$ with the usual coordinates (x, y, z). Taking the differential of the symmetric polynomials $s_1 = x + y + z$, $s_2 = xy + yz + zx$ and $s_3 = xyz$, respectively, we obtain the differential $1-$forms

$$\begin{aligned} \omega_1 &= dx + dy + dz \\ \omega_2 &= (y + z)\, dx + (z + x)\, dy + (x + y)\, dz \\ \omega_3 &= yzdx + zxdy + xydz. \end{aligned} \tag{5}$$

Computing the resulting determinant, we see that the $1-$forms ω_i, $i = 1, 2, 3$, are functionally dependent on the set S, defined by the equation $(x - y)(y - z)(z - x) = 0$, and functionally independent elsewhere *(i.e. regular points)*. Consider now the vector fields which span locally, at regular points, the pairwise intersections of kernels of the above $1-$forms, i.e.,

$$\begin{aligned} \ker\omega_2 \cap \ker\omega_3 &= span\{X_1\}, \\ \ker\omega_3 \cap \ker\omega_1 &= span\{X_2\}, \\ \ker\omega_1 \cap \ker\omega_2 &= span\{X_3\}. \end{aligned} \tag{6}$$

Our goal in what follows is to prove the stochastic connectivity property with respect to the almost-Riemannian structure induced on $\mathbb{R}^2$ by the distribution $\mathcal{D} = span\{X_1, X_2, X_3\}$.

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Stability analysis of some equilibrium points in a complex model for cells evolution in leukemia

BORDEI Ana-Maria, BADRALEXI Irina, HALANAY Andrei

The analysis regarding the stability of some equilibrium points in a complex model considers the competition between the populations of healthy and leukemic cells, the asymmetric division and the immune system in response to the disease. Delay differential equations are used to describe the dynamics of healthy and leukemic cells in case of CML (Chronic Myeloid Leukemia). The system consists of 9 delay differential equations, the first equations from 1 to 4 describe the hematopoietic healthy and leukemic cells evolution, equations 5 – 9 describe the evolution of the immune cell populations involved in the immune response against CML. The system has four possible types of equilibrium points, denoted E1, E2, E3 and E4. The study is focused on the situations E3, when leukemia cells have entirely replaced the healthy ones and E4, representing a chronic phase of the disease.