II.2. DETERMINATION OF THE ELECTRON SPECIFIC CHARGE BY USING LANGMUIR-CHILD'S 3/2 LAW FOR VACUUM DIODES

1. Work purpose

The study of the current-voltage characteristic for a vacuum diode (3/2 law) and the determination of the electron specific charge.

2. Theory

The current-voltage characteristic for a vacuum diode will be studied. Experimentally, this characteristic looks like in the graph below. $T_1 < T_2 < T_3 < T_4$ are the emissive filament (cathode) temperatures.

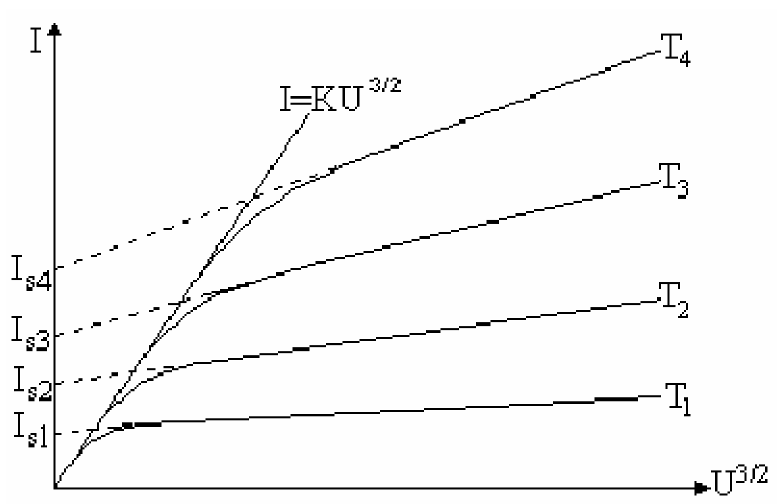

Figure 1.

At low voltages, the characteristics are of the form

$$I = KU^{3/2} \tag{1}$$

(Langmuir-Child's 3/2 law), the constant K, called perveance, being independent on the cathode temperature. At high voltages, we have

$$I = I_s \exp\left(a\sqrt{U}\right), \qquad I_s = AT^2 \exp\left(-\frac{W}{k_B T}\right) \tag{2}$$

(Richardson – Dushman's law), where I_s is the saturation current, T is the cathode temperature and W is the extraction work of the electrons from the cathode. A is a physical constant whose unit is $[A]_{SI} = [I]\cdot[T]^{-2} = A\cdot K^{-2}$. The term $\exp\left(a\sqrt{U}\right)$, slightly larger than the unit, corresponds to a decrease in the extraction work of the electrons when the voltage U is very great.

Let us deduce the equation $I = KU^{3/2}$ and compute the perveance. We shall suppose that the cathode and the anode are two coaxial electrodes of length l and radii $R_K << R_A$ (see Figure 2).

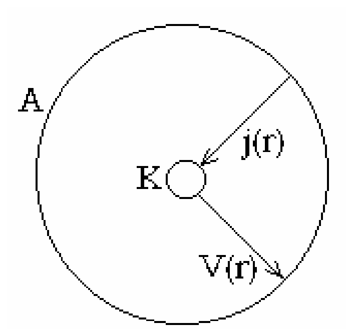

Figure 2.

At low voltages, the current is much smaller than the saturation value and many of the emitted electrons move around the cathode, forming a space charge region. We will use cylindrical coordinates (r, θ, z). If $l >> R_A$, all the physical quantities varying inside the diode depend on r only.

The equations describing the phenomenon are the flow equation

$$j(r) = \rho(r)v(r) < 0\,, \tag{3}$$

the energy conservation equation

$$\frac{m}{2}v^2(r) = eV(r) \tag{4}$$

and the Poisson equation

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dV}{dr}\right)=-\frac{\rho}{\varepsilon_0}. \tag{5}$$

Due to the diode geometry, we have

$$j(r)=-\frac{I}{2\pi lr}. \tag{6}$$

From the above equations we obtain then

$$\frac{d}{dr}\left(r\frac{dV}{dr}\right)=\frac{A}{\sqrt{V}}, \qquad A=\frac{I}{2\sqrt{2}\pi\varepsilon_0 l\sqrt{\frac{e}{m}}}. \tag{7}$$

Such an equation admits a solution of the form

$$V=Br^{\alpha}. \tag{8}$$

Replacing it into the relation (7), we get

$$\alpha^2 Br^{\alpha-1}=\frac{A}{\sqrt{B}}r^{-\frac{\alpha}{2}}, \tag{9}$$

so that α = 3/2 and

$$B^{3/2}=\frac{9I}{2\sqrt{2}\cdot 4\pi\varepsilon_0\cdot l\cdot\sqrt{\frac{e}{m}}}. \tag{10}$$

The voltage applied to the diode is

$$U=V(R_A)-V(R_K)=BR_A^{2/3}\left[1-\left(\frac{R_K}{R_A}\right)^{2/3}\right]. \tag{11}$$

From (10) and (11) it results that

$$I\,\frac{2\sqrt{2}}{9}\cdot 4\pi\varepsilon_0\cdot\frac{l}{R_A}\left[1-\left(\frac{R_K}{R_A}\right)^{2/3}\right]^{-3/2}\cdot\sqrt{\frac{e}{m}}\times U^{3/2}. \tag{12}$$

If the anode radius is not small enough as compared to its length, the cylindrical symmetry is not exact and we must introduce a correction factor

β^2. The non-zero cathode radius correction, $\left[1-\left(\frac{R_K}{R_A}\right)^{2/3}\right]^{-3/2}$, is also included in β^2. Therefore, we can write the 3/2 law in the form

$$I = KU^{3/2}, \qquad K = \frac{2\sqrt{2}}{9}\cdot 4\pi\varepsilon_0 \cdot \frac{l}{R_A\beta^2}\cdot\sqrt{\frac{e}{m}}. \qquad (13)$$

3. Experimental set-up

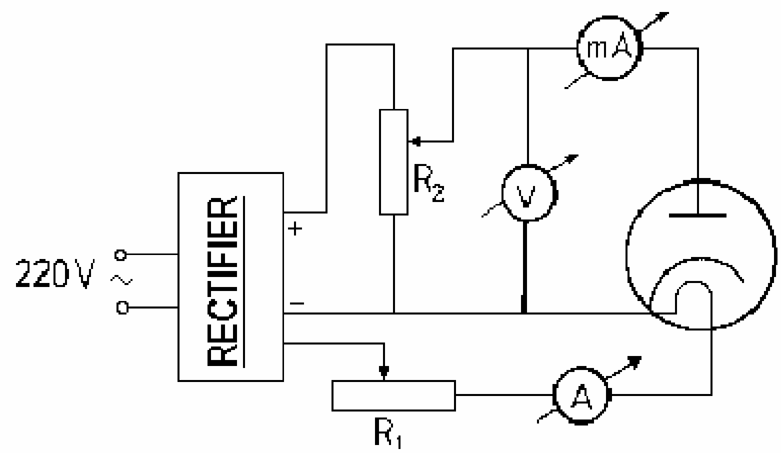

Figure 3.

The experimental set-up is presented in Figure 3. The potentiometers R_1 and R_2 allow the adjustment of the filament current and of the constant anodic voltage, respectively.

4. Working procedure

1. By using the rheostat R_1, fix through the filament the maximum current allowed by the source (usually about 70 mA).
2. By using the rheostat R_2, vary U in steps of 10 V from zero up to the superior limit allowed by the source (or by the measuring devices) and measure $I(U)$.
3. Repeat the determination of $I(U)$ for other three values of the filament current (for example 65, 60, 55 mA).

5. Experimental data processing

1. Plot on millimeter paper with coordinates I and $U^{3/2}$, the 4 current-voltage curves for the 4 filament current values.
2. Plot the common tangent in the origin of these graphs. This will be a straight line of equation $I=K_{exp}U^{3/2}$. Determine K_{exp}. (see Fig. 1). If the tangents are not common, determine K_{exp} for each curve.
3. From K_{exp}, determine the electron specific charge $(e/m)_{\mathrm{exp}}$, by using the equation

$$\left(\frac{e}{m}\right)_{exp} = \left(\frac{K_{exp}}{\frac{2\sqrt{2}}{9}\cdot 4\pi\varepsilon_0 \cdot \frac{l}{R_A}\beta^2}\right)^2 \cong \left(\frac{R_A\beta^2 K_{exp}}{3.50\cdot 10^{-11}\cdot l}\right) \text{(SI units)} \quad (14)$$

The values for l, R_A and β are given on the desk. In most cases, l = 0.95 cm, R_A = 0.55 cm and β = 1.1.

4. Compute the mean value and the standard error for the specific charge.

Observation: The electron specific charge is about $1.76\cdot 10^{11}$ C/kg. If your result is strongly different from this value, find out the reason.

6. Questions

1. How many distinct zones does the current-voltage curve of a diode have and what laws can be applied to them?
2. Why doesn't the expression of the 3/2 law contain the temperature of the filament, the current of the diode being independent on it?
3. What fundamental equations are used for deducing the 3/2 law?
4. What approximations have been made during the computations?