III.7. THE STUDY OF THE HALL EFFECT IN SEMICONDUCTORS

1. Work purpose

The Hall effect is one of the most important effects in the determination of the parameters that characterize from the electrical point of view the semiconductor materials. The goals of the work are:

- The determination of the concentration of the charge carriers (n or p) in a sample of extrinsic semiconductors*;
- The determination of the Hall mobility of the charge carriers in the respective semiconductor.

2. Theory

The Hall effect is a galvanomagnetic** effect, which was observed for the first time by E. H. Hall in 1880. This effect consists in the appearance of an electric field called Hall field $\vec{E}_H$, due to the deviation of the charge carrier trajectories by an external magnetic field. We will study the Hall effect in a parallelepipedic semiconductor sample of sizes *a, b, c* (see Figure 1). The Hall field appears when the sample is placed under an external electric field $\vec{E}$ and an external magnetic field $\vec{B}$. The Hall field $\vec{E}_H$ is orthogonal on both $\vec{E}$ and $\vec{B}$. The vectors $\vec{E}$, $\vec{E}_H$ and $\vec{B}$ determine a right orhogonal trihedron (Figure 1):

$$\vec{E} = (E, 0, 0), \quad \vec{E}_H = (0, E_H, 0), \quad \vec{B} = (0, 0, B). \tag{1}$$

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* Extrinsic semiconductors are semiconductors with impurities in which the electric conduction is done either by electrons (semiconductors with donor impurities called n-type semiconductors), or by holes (semiconductors with acceptor impurities called p-type semiconductors).

** Galvanomagnetic effects are physical phenomena that appear in substances during the interaction between the externally applied magnetic field and the charges moving through those substances.

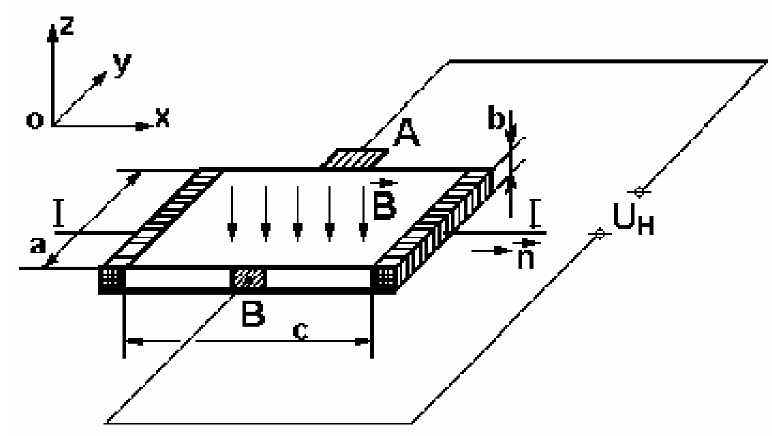

Figure 1.

Under the action of the external electric field $\vec{E} = (E, 0, 0)$, through the semiconductor sample flows a current *I*. Applying on the sample the magnetic field $\vec{B} = (0, 0, B)$, a potential difference U_H, called Hall bias, appears between its lateral faces, on the direction normal to both $\vec{E}$ and $\vec{B}$:

$$U_H = V_A - V_B. \tag{2}$$

The Hall bias is determined by the deviation of the charge carriers, which form a current through the sample, under the action of the Lorenz force:

$$\vec{F}_L = \pm e(\vec{v} \wedge \vec{B}), \tag{3}$$

where $\vec{v}$ is the average (drift) velocity of the charge carriers moving through the sample under the action of the field $\vec{E}$ and *e* is the elementary charge ($e \cong 1.6 \cdot 10^{-19}$ C). The absolute value of the Hall field intensity is:

$$E_H = \frac{U_H}{a}. \tag{4}$$

The external and the Hall electric fields produce the electric force $\vec{F}_{el}$:

$$\vec{F}_{el} = e\vec{E}_t = e(\vec{E} + \vec{E}_H), \tag{5}$$

where $\vec{E}_t$ is the total electric field. The total force that acts on the charge carriers is:

$$\vec{F}_t = \vec{F}_{el} + \vec{F}_L. \tag{6}$$

The (negative) electrons and (positive) holes moving through the sample satisfy the equations:

$$\frac{d\vec{v}_e}{dt} + \frac{\vec{v}_e}{\tau_e} = -\frac{e}{m_e^*}\left(\vec{E}_t + \vec{v}_e \wedge \vec{B}\right), \tag{7}$$

$$\frac{d\vec{v}_h}{dt} + \frac{\vec{v}_h}{\tau_h} = \frac{e}{m_h^*}\left(\vec{E}_t + \vec{v}_h \wedge \vec{B}\right), \tag{8}$$

where m_e^*, m_h^* and τ_e, τ_h are the electrons and holes effective masses and relaxation times, respectively. In steady states, the time derivatives cancel and we will define the electron and hole mobilities as:

$$\mu_n = \frac{e\tau_e}{m_e^*}, \quad \mu_p = \frac{e\tau_h}{m_h^*}, \tag{9}$$

so that Eqs. (7), (8) become:

$$\begin{cases} v_{ex} = -\mu_n\left(E + v_{ey}B\right), \\ v_{ey} = -\mu_n\left(E_H - v_{ex}B\right), \\ v_{ez} = 0 \end{cases} \tag{7'}$$

and:

$$\begin{cases} v_{hx} = \mu_p\left(E + v_{hy}B\right), \\ v_{hy} = \mu_p\left(E_H - v_{hx}B\right), \\ v_{hz} = 0. \end{cases} \tag{8'}$$

The current density is defined as:

$$\vec{j} = e\left(p\,\vec{v}_h - n\,\vec{v}_e\right), \tag{10}$$

where n and p are the electron and hole concentrations. Solving the equations (6 – 10) under the condition $j_y = 0$ ($\vec{j} = (j, 0, 0)$), we obtain:

$$j = \sigma(B)E, \tag{11}$$

$$E_H = R_H(B)jB, \tag{12}$$

where the conductivity σ and the Hall constant R_H are given by the relations:

$$\sigma(B) = e(n\mu_n + p\mu_p)\left[1 + \frac{np\mu_n\mu_p(\mu_p - \mu_n)^2 B^2}{(n\mu_n + p\mu_p)^2 + (p-n)^2\mu_n^2\mu_p^2 B^2}\right]^{-1}, \qquad (13)$$

$$R_H(B) = \frac{1}{e}\cdot\frac{p\mu_p^2 - n\mu_n^2 + (p-n)\mu_n^2\mu_p^2 B^2}{(n\mu_n + p\mu_p)^2 + (p-n)^2\mu_n^2\mu_p^2 B^2}. \qquad (14)$$

One can observe that the conductivity monotonously decreases from:

$$\sigma(0) \equiv \sigma_0 = e(n\mu_n + p\mu_p) \qquad (15)$$

to

$$\sigma(\infty) \equiv \sigma_\infty = \sigma_0\left[1 + \frac{np(\mu_p - \mu_n)^2}{(p-n)^2\mu_n\mu_p}\right]^{-1}, \qquad (16)$$

while the Hall constant monotonously increases from:

$$R_H(0) \equiv R_{H0} = \frac{1}{e}\frac{p\mu_p^2 - n\mu_n^2}{(n\mu_n + p\mu_p)^2} \qquad (17)$$

to:

$$R_H(\infty) \equiv R_{H\infty} = \frac{1}{e(p-n)}. \qquad (18)$$

For intrinsic semiconductors ($n = p \equiv n_i$), we have $\sigma_\infty = 0$ and:

$$R_H(B) = \frac{1}{en_i}\cdot\frac{\mu_p - \mu_n}{\mu_p + \mu_n}. \qquad (19)$$

For heavily doped (extrinsic) semiconductors we have:

$$\sigma(B) \cong en\mu_n, \quad R_H(B) \cong -\frac{1}{en}, \quad n >> p, \qquad (20)$$

$$\sigma(B) \cong ep\mu_p, \quad R_H(B) \cong \frac{1}{ep}, \quad p >> n. \qquad (21)$$

From these relations, one can observe that the Hall mobility of the carriers can be defined as:

$$\mu_H(B) = \sigma(B)|R_H(B)|. \tag{22}$$

For our sample geometry, we obtain from Eqs. (4) and (12):

$$|R_H| = \frac{U_H}{ajB} = \frac{bU_H}{iB}, \tag{23}$$

where i is the current flowing in *Ox* direction and:

$$\sigma = \frac{ab}{cr_x} = \frac{cb}{ar_y}, \tag{24}$$

r_x, r_y being the sample resistance in *Ox*, *Oy* directions, respectively.

One can observe that the units for the Hall constant and mobility are:

$$\langle R_H \rangle_{IS} = \left\langle \frac{1}{en} \right\rangle_{IS} = m^3C^{-1}, \quad \langle \mu_H \rangle_{IS} = \left\langle \frac{v}{e} \right\rangle_{IS} = m^2V^{-1}s^{-1} = T^{-1}. \tag{25}$$

3. Experimental set-up

The sample set-up presented in Figure 2, is made of:

- an electromagnet with weak magnetic remnant steel core, which allows a better concentration of the magnetic field lines;
- a box containing the p-type semiconductor sample that is studied.

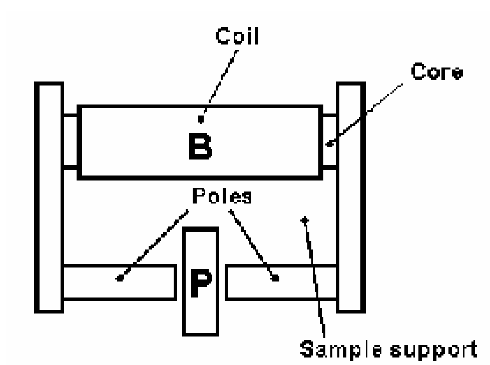

Figure 2.

In Figure 3 we have the electric draft of the measuring circuits. In Figure 3a is given the draft for the measurement of the current through the sample (for different values of the d. c. bias applied on the sample) and of the Hall voltage. In Figure 3b is given the draft of the electromagnet circuit.

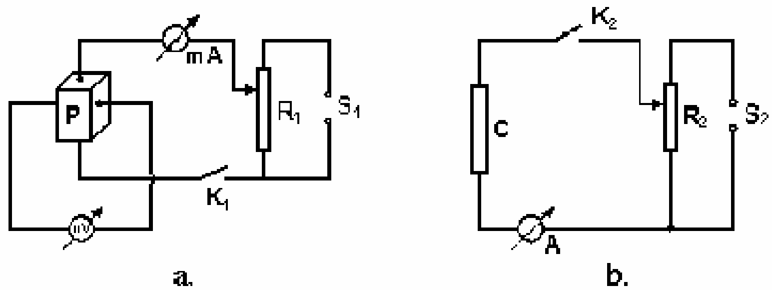

Figure 3.

Figure 3a presents the circuit that supplies the sample, including the sample P, a milliammeter mA, to measure the current i through the sample, a millivoltmeter mV, to measure the Hall voltage (using the compensation method we obtain a more accurate measure of U_H), the source S_1, the potentiometer R_1, and the switch K_1.

Figure 3b presents the electromagnet circuit, including the electromagnet coil C, the ammeter A to measure the current I through the electromagnet, the potentiometer R_2, the source S_2, and the switch K_2.

4. Working procedure:

1. Connect the sources S_1 and S_2 by plugging them and turning on the switches K_1 and K_2. Using the potentiometer R_1 a constant current i through the sample is fixed. The values of the current i for which the measurements have to be performed are indicated on the desk.
2. Using the potentiometer R_2, vary the current I through the electromagnet coil from 0.2 A to the maximum value of 3A.
3. For each value of I, read U_H using the millivoltmeter mV.
4. Repeat the measurements for different values of the current i through the sample. The results will be written in Table 1, which contains the values of the induction B for different values of the current I through the coil. The sample sizes *a, b, c,* are given on the desk. The current density j is:

$$j = \frac{i}{ab}. \tag{26}$$

Table 1

U_H (V)	i (mA)	...
I (A)	j (A/m^2) / B (T)	...
0.0	0.016	
0.2	0.040	
0.4	0.068	
0.6	0.098	
0.8	0.128	
1.0	0.156	
1.2	0.170	
1.4	0.180	
1.6	0.190	
1.8	0.196	
2.0	0.202	
2.2	0.208	
2.4	0.213	
2.6	0.218	
2.8	0.222	
3.0	0.225	

5. Draw the electromagnet calibration curve $B = B(I)$, in order to find the values of B for the coil current values different from those in Table 1. As when the current is flowing through both sample and coil these devices can go out of order, turn on the switches K_1 and K_2 only when we read the data.

5. Experimental data processing

1. Using the data from Table 1, plot on millimetric paper the dependencies $U_H = U_H(B)$ for $j = const$. Due to the non-linearity of the Hall effect and the sample heating during the measurements, we will not obtain straight lines; the tangents to the curves at the origin (B = 0) will have the slopes:

$$m_k = aR_H j_k, \quad k = 1, 2, ..., N. \tag{27}$$

With the slopes m_1, m_2, ... m_N, obtained from the graphs, compute the values of the corresponding Hall constants:

$$R_H^{(k)} = \frac{m_k}{a j_k}. \tag{28}$$

Compute the mean Hall constant:

$$\langle R_H \rangle = \frac{1}{N} \sum_{k=1}^{N} R_H^{(k)}. \tag{29}$$

The value of the Hall constant will be given in the form:

$$R_H = \langle R_H \rangle \pm \sigma_{\langle R_H \rangle}, \tag{30}$$

where σ_{R_H} represents the mean square deviation of the mean Hall constant, which is computed with the formula:

$$\sigma_{R_H} = \sqrt{\frac{1}{N(N-1)} \sum_{k=1}^{N} \left(R_H^{(k)} - \langle R_H \rangle \right)^2}. \tag{31}$$

2. Knowing the values of the Hall constant, the mean charge carrier (hole) concentration for the sample is determined using the relation (21):

$$\langle p \rangle = \frac{1}{\langle R_H \rangle e}. \tag{32}$$

The Hall mobility is determined from the relations (22), (24). The errors for p and μ_H are computed taking into account that the other constants are supposed exact, so that the relative errors for p, μ_H and R_H are equal.

When processing the experimental data, all physical quantities are expressed in I. S. units.

6. Questions:

1. What means the Hall effect?
2. What is the Hall constant?
3. What is the Hall mobility?