III.2. ELECTRIC FIELD INFLUENCE UPON THE THERMOELECTRONIC EMISION

1. Purpose:

The determination of the extraction work of the free electrons from a metal using the thermoelectronic emission phenomenon and the study of electronic emission under the influence of intense electric fields (Schottky effect).

2. Theory:

2. A. *Richardson-Dushman law*

In a vacuum diode, the current – voltage characteristic presents distinguished regions depending on the applied acceleration voltage. When the anode voltage is high enough to attract all the electrons of the space charge region from the cathode, we have a saturation current independent of the applied voltage. The saturation value depends on the filament heating voltage, meaning the value of the supplementary thermal energy given to the metal electrons. The dependence of the saturation thermoelectric current density on the heating temperature of the metal, known as the Richardson-Dushman law, is:

$$j = AT^2 \exp\left(-\frac{B}{T}\right), \tag{1}$$

where A and B are constant values for a given metal and T is the absolute temperature.

The study of the electronic emission phenomenon can be conveniently made using the Sommerfeld model, where the electron potential energy value is considered constant and negative inside the metal, $E_p \equiv -W = const < 0$.

Because outside the material $W = 0$, in this model the metal can be described as a finite potential well. When $T = 0$ K, the quasifree electrons occupy the energetic levels beginning with the lowest energy until they reach a maximum energy equal to the Fermi energy ε_F, on each level being two electrons with opposite spins, in agreement with the Pauli exclusion law (see Figure 1).

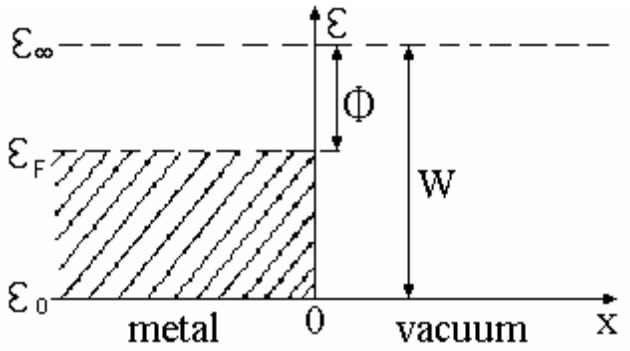


Figure 1.

Let W be the necessary work for a free electron of a metal that is in the lowest energetic state ε_0 to leave the surface of the metal and go in vacuum to an infinite distance ε_∞ (this energetic level is called vacuum level), $W = \varepsilon_\infty - \varepsilon_0$. The difference between the vacuum level and the Fermi level $\Phi = W - \varepsilon_F$ is called the extraction work and represents the necessary work for a quasifree electron of a metal situated on the Fermi level to leave the metal and become free.

To compute the emission current density, meaning the electronic flow that leaves the surface of the metal during the time unit in the direction *Ox*, we will consider that all the electrons leaving the metal have the momentum component p_x such that:

$$\frac{p_x^2}{2m} \geq W \,. \tag{2}$$

Let *dn* be the number of electrons from the unit volume having the momentum components $p_x \in [p_x, p_x + dp_x]$, $p_y \in \lfloor p_y, p_y + dp_y \rfloor$ and $p_z \in [p_z, p_z + dp_z]$:

$$dn = \frac{2}{h^3} f(\varepsilon) dp_x dp_y dp_z, \tag{3}$$

where $f(\varepsilon)$ is the Fermi-Dirac distribution function. Then, the number of electrons from the unit volume with p_x in the interval $[p_x, p_x + dp_x]$, not taking into consideration p_y and p_z is:

$$dn_x = \frac{2}{h^3} dp_x \left\{ \int_{-\infty}^{\infty} dp_y \int_{-\infty}^{\infty} dp_z \left[\exp\left(\frac{\varepsilon - \varepsilon_F}{k_B T} \right) + 1 \right]^{-1} \right\}. \tag{4}$$

Taking into account that the thermoelectric emission phenomena takes place at high temperatures, we can approximate $f(\varepsilon)$ with the Maxwell-Boltzmann distribution function. Replacing $\varepsilon = \frac{p_x^2 + p_y^2 + p_z^2}{2m}$, it results:

$$f(\varepsilon) \approx \exp\left(\frac{\varepsilon_F}{k_B T} \right) \exp\left(-\frac{p_x^2 + p_y^2 + p_z^2}{2mk_B T} \right). \tag{5}$$

Replacing (5) in (4) we have:

$$dn_x = \frac{2}{h^3} \exp\left(\frac{\varepsilon_F}{k_B T} \right) \exp\left(-\frac{p_x^2}{2mk_B T} \right) dp_x$$

$$\times \int_{-\infty}^{\infty} \exp\left(-\frac{p_y^2}{2mk_B T} \right) dp_y \int_{-\infty}^{\infty} \exp\left(-\frac{p_z^2}{2mk_B T} \right) dp_z$$

$$= \frac{4\pi mk_B T}{h^3} \exp\left(\frac{\varepsilon_F}{kT} \right) \cdot \exp\left(-\frac{p_x^2}{2mk_B T} \right) dp_x. \tag{6}$$

But:

$$dj_x = ev_x dn_x, \tag{7}$$

so that:

$$j_x = \int dj_x = \frac{4\pi e k_B T}{h^3} \exp\left(\frac{\varepsilon_F}{k_B T}\right) \int_{\sqrt{2mW}}^{\infty} p_x \exp\left(-\frac{p_x^2}{2mk_B T}\right) dp_x . \tag{8}$$

We have then:

$$j = \frac{8\pi mek_B^2}{h^3} T^2 \exp\left(-\frac{\Phi}{k_B T}\right) \equiv A_0 T^2 e^{-\frac{B}{T}} . \tag{9}$$

The Richardson-Dushman law (9) is in good agreement with the experimental results.

However, at the quantitative evaluations, there are some discrepancies. These are the result of many causes, such as:

a) The extraction work depends on the temperature $\Phi = \Phi(T)$. Because of the variation of $W(T)$ and $\varepsilon_F(T)$ with the temperature, the extraction work in different metals can increase or decrease with the increase of the temperature. However, this variation is weak and, at first approximation, we can consider a linear dependence having the form: $\Phi(T) = \Phi(T_0) \pm \alpha(T - T_0)$, where α is a constant of the order of 10^{-4} eV/K. Quoting: $A = A_0 \exp(\pm \alpha / k_B)$, we can write the thermoelectric current temperature dependence like in the relation (1). By extracting a logarithm from the relation (1) we obtain:

$$\ln \frac{j}{T^2} = \ln A - \frac{B}{T} = \ln A - \frac{\Phi}{k_B T}, \tag{13}$$

used in experiments. The quantities □ and *A* are called reduced extraction work and Richardson-Dushman thermoelectric constant, respectively.

b) The extraction work also depends on the properties of the emissive substance. For instance, the electronic affinity depends on the crystallographic direction and on the arrangement of the surface

atoms. This arrangement can be modified due to impurity absorption. Due to this, the exact experimental information concerning the themoelectronic emission needs to specify the crystallographic direction and to perform the measurements in high vacuum (10^{-9} torr) and on metals previously well degassed. The value of the extraction work for the metals is of a few electronvolts only.

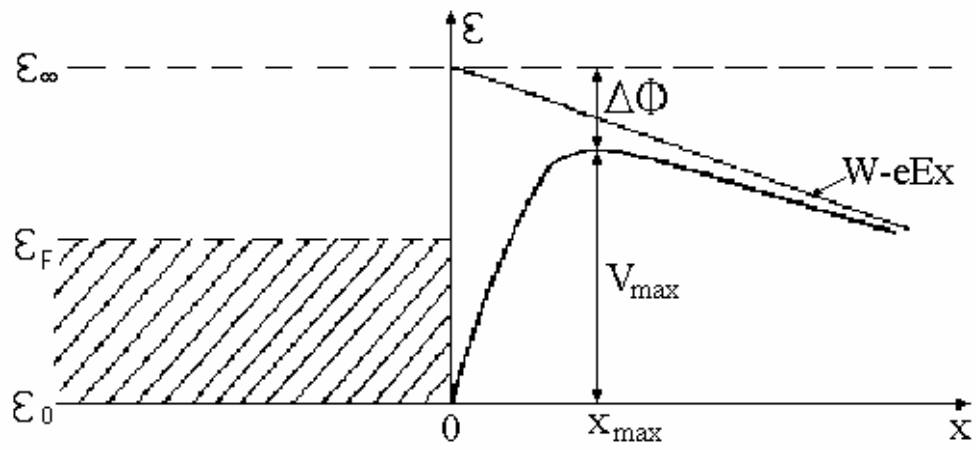
2. B. *Schotty Effect*

The experimental results proved that, for metals, the thermoelectronic emission current does not have a real saturation, but it keeps increasing with the increasing of the applied electric field. Beginning with a certain applied voltage, this increase becomes important and the current-voltage characteristic of the thermoelectronic emission does not present any saturation. This phenomenon of increase of the thermoelectronic emission current when applying an electric field is called Schottky effect. Its explanation can also be given using the Sommerfeld model.

By applying an external electric field $\vec{E}$, one creates an additional force $\vec{F} = -e\vec{E}$, opposed to the image charge force. In this case, the shape of the potential well is modified and the electron potential energy will be:

$$V(x) = \begin{cases} 0, & x < 0 \\ W - \dfrac{e^2}{16\pi\varepsilon_0 x} - eEx, & x > 0 \end{cases} \tag{15}$$

The shape of the potential energy (that is of the potential well) for



$x > 0$ is the result of action of both the image charge force and the external field. In Figure 2 is presented the electron potential energy at the metal – vacuum frontier (the straight line with negative slope represents the electron potential energy in the presence of only the external electric field).

Figure 2.

The electron potential energy presents a maximum, which is determined from the condition $\frac{dV}{dx} = 0$ and which appears at the distance:

$$x_{\max} = \frac{1}{4}\sqrt{\frac{e}{\pi \varepsilon_0 E}} \tag{16}$$

from the metal surface. This represents the distance at which the external field cancels the action of the image charge force. The potential energy value in this point is:

$$V_{\max} \equiv V\left(x_{\max}\right) = W - \frac{1}{2}\sqrt{\frac{e^3 E}{\pi \varepsilon_0}}\,. \tag{17}$$

The extraction work depends on field by means of $V_{\max}$ through the relation:

$$V_{\max} = \varepsilon_F + \Phi(E), \tag{18}$$

while in the absence of the field ($\Phi(0) \equiv \Phi_0$):

$$V_\infty \equiv W = \varepsilon_F + \Phi_0\,. \tag{19}$$

From the relations (18) and (19) we obtain:

$$\Phi(E) = \Phi_0 - \frac{1}{2}\sqrt{\frac{e^3 E}{\pi \varepsilon_0}} \equiv \Phi_0 - \Delta\Phi\,, \tag{20}$$

meaning that, under the influence of the external field, the extraction work decreases. Then, the emission current increases with the electric field and we obtain:

$$j(T,E)=j(T,0)\exp\left(\frac{1}{2k_BT}\sqrt{\frac{e^3E}{\pi\varepsilon_0}}\right), \tag{21}$$

where $j(T,0)$ is the thermoelectric current density in the absence of the electric field. By extracting the logarithm from the relation (21), we obtain:

$$\log\frac{j(T,E)}{j(T,0)}=\frac{1}{2k_BT}\sqrt{\frac{e^3}{\pi\varepsilon_0}}\cdot E^{1/2}. \tag{22}$$

We observe that the diagram $\log j(T,E)/j(T,0)=f\left(E^{1/2}\right)$ is a straight line, whose slope is directly proportional to 1/T. In Figure 3 is presented the Schottky emission current as a function of the applied field for tungsten at T = 1373 K. The continuous line represents the theoretical line, in conformity to relation (22). The theoretical dependence is satisfactorily confirmed by the experimental data obtained for pure metal. By drawing the mentioned dependence for different temperatures, we obtain a family of lines whose slopes are inversely proportional to the temperatures. Usually, there are some displacements from this pattern in the regions where the field is small or very high. In the first case the influence of the contact field at the emitter surface appears. In the second case we have the influence of the transmission coefficient, which is not only a function of the electron energy but also of the electric field.

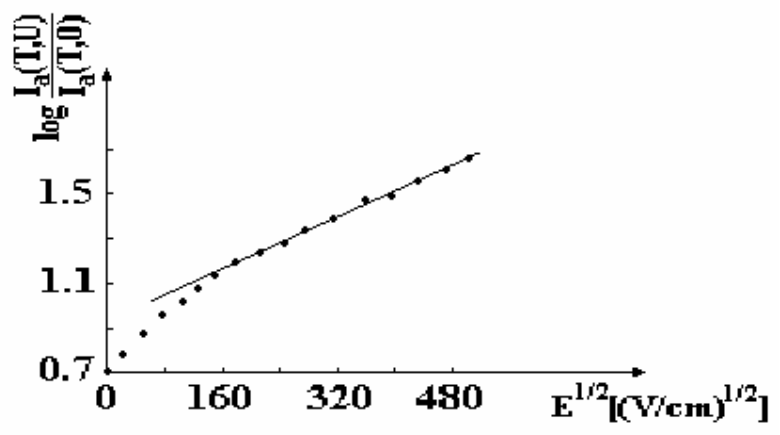


Figure 3.

In this work the linear dependence of the function $\log[I_a(T,U)/I_a(T,0)] = f\left(U^{1/2}\right)$ for different cathode temperatures is verified.

3. Experimental set-up

The draft of the experimental set-up is presented in Figure 4.

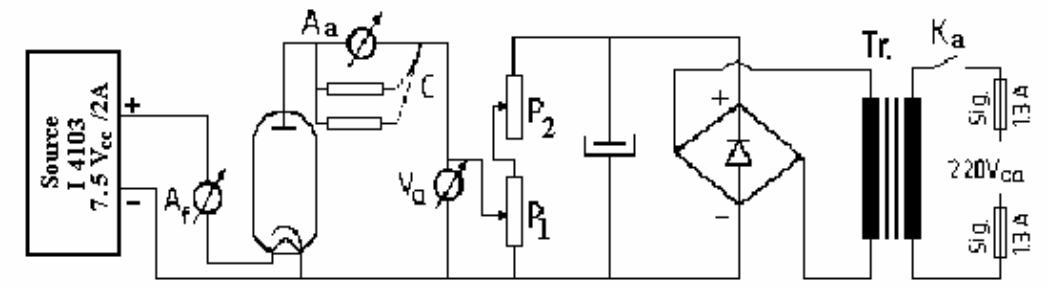


Figure 4.

4. Working procedure

We measure the anode current I_a with respect to the anode voltage U_a for different values of the filament current I_f.

a) Filament circuit

1. We turn on the 7.5 V source. We adjust the voltage using the potentiometer U_f (with the switch fixed on the 7.5 V position), for the first value of the filament current read on the miliammeter A_f on a scale of 0.24 A.

2. The filament current values are adjusted from division to division from 54 mA to 64 mA. The variation of the cathode temperature depending on the filament current is given in Table 1. **Observation:** After each adjustment of the filament current, we must wait 5 minutes for the reaching of the thermal equilibrium between filament, cathode and the electronic gas.

Table 1

I_f (mA)	54	56	58	60	62	64
T (K)	1013	1045	1074	1106	1138	1172

Table 2

I_f (mA) / U_a (V)	54	56	58	60	62	64
0						
20						
40						
60						
80						
100						
120						
140						
160						
180						
200						

b) Anode circuit

1. We start the anode circuit with the switch K_a and we verify if the anode voltage is zero.
2. We increase the anode voltage in steps of 20V. The coarse adjustment of the voltage is made using the potentiometer P_1 and the fine adjustment is made using the potentiometer P_2 until it reaches 120V. **Observation:** For $I_f = 58$ mA, see point 3.

3. If the value of the filament voltage is of 58 mA, we determine the dependence between the anode current and the anode voltage until the anode voltage reaches 200V. The obtained experimental data are used to highlight the Schottky effect. (See next Section, point II).
4. After the measurements are complete, we reduce all the voltages to zero using the corresponding potentiometers and we switch off the circuits. The measured values for I_a (expressed in mA) are written in Table 2.

5. Experimental data processing

I. Richardson – Dushman law

1. We study the dependence between the anode current I_a and the anode voltage U_a for different values of the filament current I_f. We will increase the anode voltage in steps of 20V, until we obtain the same (saturation) current value for I_f = 54, 56, 60, 62, and 64 mA.
2. We determine the cathode temperature corresponding to the filament current from the Table 1.
3. We draw the anodic current – voltage characteristic for the different values of the filament current.
4. We draw the graph $\log(I_s / T^2) = f(1/T)$.
5. We compute from the graph the slope of the straight line $\log(I_s / T^2) = \log A - B/T$, where $B = \phi / k_B$ and k_B = 8.62 · 10^{-5} eV/K. We determine the reduced extraction work $\phi = B\,k_B$ (eV) for the cathode, which is made of a Ni alloy activated with Ba and Sr oxides.

To determine the slope of the straight line $\log(I_s / T^2) = f(1/T)$, we can also apply the method of the least squares. Let $y = ax + b$ the line equation. If $y = \log\left(I_s / T^2\right)$, $x = 1/T$, $a = -B$, and $b = \log A$, the estimates a^* and b^* of the constants a and b are given by the relations:

$$a^* = \frac{n\sum_{i=1}^{n} x_i y_i - (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2},\ b^* = \frac{\sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} y - (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} x_i y_i)}{n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2}, \quad (23)$$

where $\{x_i, y_i\}, i = \overline{1, n}$ are the n pairs of the experimentally measured values. The estimates of the *a, b* and *y* dispersions are:

$$s_{a^*}^2 = \frac{n}{n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2} s_y^2, \qquad s_{b^*}^2 = \frac{\sum_{i=1}^{n} x_i^2}{n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2} s_y^2, \quad (24)$$

where

$$s_y^2 = \frac{\sum_{i=1}^{n} (y_i - b^* - a^* x_i)^2}{n(n-1)}. \quad (25)$$

Then B will have the final formula $B = B^* \pm s_{B^*}$ and the reduced extraction work $\Phi = k_B B$ is $\Phi = \Phi^* \pm s_{\Phi^*}$.

II. Schotty Effect

1. We verify the linear dependence of $\log j/j_0 = f\left(E^{1/2}\right)$ – given by the equation (22) – by plotting the experimental data obtained at the point 3, namely $\log I_a / I_{a0} = f\left(U_a^{1/2}\right)$, for a filament current I_f = 58 mA.

6. Questions

1. What is the physical meaning of the free electron extraction work of a metal and what are the physical parameters that it depends on?
2. Why the Schottky effect is also called “emission at cool”?