

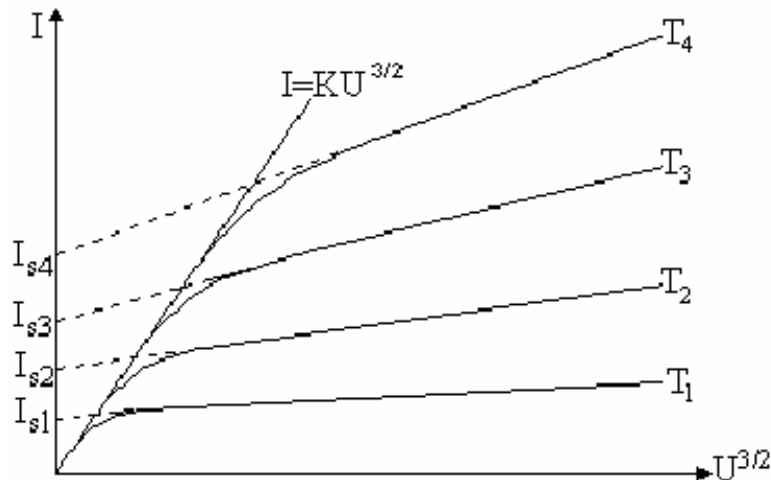
## II.2. DETERMINATION OF THE ELECTRON SPECIFIC CHARGE BY USING LANGMUIR-CHILD'S 3/2 LAW FOR VACUUM DIODES

### 1. Work purpose

The study of the current-voltage characteristic for a vacuum diode (3/2 law) and the determination of the electron specific charge.

### 2. Theory

The current-voltage characteristic for a vacuum diode will be studied. Experimentally, this characteristic looks like in the graph below.  $T_1 < T_2 < T_3 < T_4$  are the emissive filament (cathode) temperatures.



**Figure 1.**

At low voltages, the characteristics are of the form

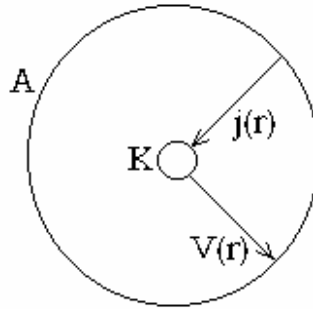
$$I = KU^{3/2} \quad (1)$$

(Langmuir-Child's 3/2 law), the constant  $K$ , called perveance, being independent on the cathode temperature. At high voltages, we have

$$I = I_s \exp(a\sqrt{U}), \quad I_s = AT^2 \exp\left(-\frac{W}{k_B T}\right) \quad (2)$$

(Richardson – Dushman's law), where  $I_s$  is the saturation current,  $T$  is the cathode temperature and  $W$  is the extraction work of the electrons from the cathode.  $A$  is a physical constant whose unit is  $[A]_{\text{SI}} = [I] \cdot [T]^{-2} = \text{A} \cdot \text{K}^{-2}$ . The term  $\exp(a\sqrt{U})$ , slightly larger than the unit, corresponds to a decrease in the extraction work of the electrons when the voltage  $U$  is very great.

Let us deduce the equation  $I = KU^{3/2}$  and compute the perveance. We shall suppose that the cathode and the anode are two coaxial electrodes of length  $l$  and radii  $R_K \ll R_A$  (see Figure 2).



**Figure 2.**

At low voltages, the current is much smaller than the saturation value and many of the emitted electrons move around the cathode, forming a space charge region. We will use cylindrical coordinates  $(r, \theta, z)$ . If  $l \gg R_A$ , all the physical quantities varying inside the diode depend on  $r$  only.

The equations describing the phenomenon are the flow equation

$$j(r) = \rho(r)v(r) < 0, \quad (3)$$

the energy conservation equation

$$\frac{m}{2} v^2(r) = eV(r) \quad (4)$$

and the Poisson equation

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{dV}{dr} \right) = -\frac{\rho}{\epsilon_0}. \quad (5)$$

Due to the diode geometry, we have

$$j(r) = -\frac{I}{2\pi lr}. \quad (6)$$

From the above equations we obtain then

$$\frac{d}{dr} \left( r \frac{dV}{dr} \right) = \frac{A}{\sqrt{V}}, \quad A = \frac{I}{2\sqrt{2}\pi\epsilon_0 l \sqrt{\frac{e}{m}}}. \quad (7)$$

Such an equation admits a solution of the form

$$V = Br^\alpha. \quad (8)$$

Replacing it into the relation (7), we get

$$\alpha^2 Br^{\alpha-1} = \frac{A}{\sqrt{B}} r^{-\frac{\alpha}{2}}, \quad (9)$$

so that  $\alpha = 3/2$  and

$$B^{3/2} = \frac{9I}{2\sqrt{2} \cdot 4\pi\epsilon_0 \cdot l \cdot \sqrt{\frac{e}{m}}}. \quad (10)$$

The voltage applied to the diode is

$$U = V(R_A) - V(R_K) = BR_A^{2/3} \left[ 1 - \left( \frac{R_K}{R_A} \right)^{2/3} \right]. \quad (11)$$

From (10) and (11) it results that

$$I \frac{2\sqrt{2}}{9} \cdot 4\pi\epsilon_0 \cdot \frac{l}{R_A} \left[ 1 - \left( \frac{R_K}{R_A} \right)^{2/3} \right]^{-3/2} \cdot \sqrt{\frac{e}{m}} \times U^{3/2}. \quad (12)$$

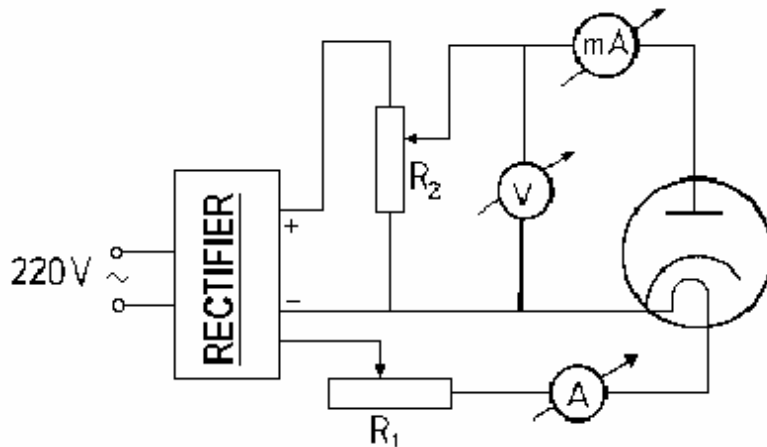
If the anode radius is not small enough as compared to its length, the cylindrical symmetry is not exact and we must introduce a correction factor

$\beta^2$ . The non-zero cathode radius correction,  $\left[1 - \left(\frac{R_K}{R_A}\right)^{2/3}\right]^{-3/2}$ , is also

included in  $\beta^2$ . Therefore, we can write the 3/2 law in the form

$$I = KU^{3/2}, \quad K = \frac{2\sqrt{2}}{9} \cdot 4\pi\epsilon_0 \cdot \frac{l}{R_A\beta^2} \cdot \sqrt{\frac{e}{m}}. \quad (13)$$

### 3. Experimental set-up



**Figure 3.**

The experimental set-up is presented in Figure 3. The potentiometers  $R_1$  and  $R_2$  allow the adjustment of the filament current and of the constant anodic voltage, respectively.

### 4. Working procedure

1. By using the rheostat  $R_1$ , fix through the filament the maximum current allowed by the source (usually about 70 mA).
2. By using the rheostat  $R_2$ , vary  $U$  in steps of 10 V from zero up to the superior limit allowed by the source (or by the measuring devices) and measure  $I(U)$ .
3. Repeat the determination of  $I(U)$  for other three values of the filament current (for example 65, 60, 55 mA).

## 5. Experimental data processing

1. Plot on millimeter paper with coordinates  $I$  and  $U^{3/2}$ , the 4 current-voltage curves for the 4 filament current values.
2. Plot the common tangent in the origin of these graphs. This will be a straight line of equation  $I=K_{exp}U^{3/2}$ . Determine  $K_{exp}$ . (see Fig. 1). If the tangents are not common, determine  $K_{exp}$  for each curve.
3. From  $K_{exp}$ , determine the electron specific charge  $(e/m)_{exp}$ , by using the equation

$$\left(\frac{e}{m}\right)_{exp} = \left( \frac{K_{exp}}{\frac{2\sqrt{2}}{9} \cdot 4\pi\epsilon_0 \cdot \frac{l}{R_A} \beta^2} \right)^2 \cong \left( \frac{R_A \beta^2 K_{exp}}{3.50 \cdot 10^{-11} \cdot l} \right) \text{ (SI units)} \quad (14)$$

The values for  $l$ ,  $R_A$  and  $\beta$  are given on the desk. In most cases,  $l = 0.95$  cm,  $R_A = 0.55$  cm and  $\beta = 1.1$ .

4. Compute the mean value and the standard error for the specific charge.

**Observation:** The electron specific charge is about  $1.76 \cdot 10^{11}$  C/kg. If your result is strongly different from this value, find out the reason.

## 6. Questions

1. How many distinct zones does the current-voltage curve of a diode have and what laws can be applied to them?
2. Why doesn't the expression of the 3/2 law contain the temperature of the filament, the current of the diode being independent on it?
3. What fundamental equations are used for deducing the 3/2 law?
4. What approximations have been made during the computations?