Determination of electron charge to mass quotient

1 Objectives

- understanding how electric and magnetic fields influence the motion of an electron beam;
- determination of the electron charge to mass quotient based on its circular motion in a uniform magnetic field.

2 Motivation of charge to mass interest

Consider a particle of mass m and electric charge q placed in an electric field of intensity $\vec{E}$ and a magnetic field of magnetic induction $\vec{B}$. The force acting on the particle is the Lorentz force

$$\vec{F}_{\mathrm{L}} = q(\vec{E} + \vec{v} \times \vec{B}), \tag{1}$$

where $\vec{v}$ is the particle velocity. Newton second law applied to the particle motion is

$$\vec{F}_{\mathrm{L}} = m\vec{a}, \tag{2}$$

where $\vec{a}$ is the particle acceleration. Combining Eqs. (1) and (2), it follows that

$$\vec{a} = (q/m)(\vec{E} + \vec{v} \times \vec{B}) \tag{3}$$

This equation reveals that two particles with the same charge to mass quotient, q/m, behave in the same way. The quantity q/m is also called *specific charge* in the literature.

The electron is a subatomic particle. Its mass is denoted m below and its electric charge is $-e$, where e is the elementary charge. The electron charge to mass quotient, $-e/m$, is a quantity that can be measured experimentally and a method is presented in this report. The determination of e can be done for example in the Millikan's oil drop experiment. Combining the results in these experiments, electron mass is calculated.

3 Theory

If an electron at rest is accelerated by a potential difference U, it attains the kinetic energy

$$(1/2)mv_0^2 = eU, \tag{4}$$

where v_0 is the modulus of the velocity after acceleration.

Suppose now the electron enters a region of uniform magnetic field of magnetic induction $\vec{B}$. The force acting upon the electron is

$$\vec{F} = -e(\vec{v} \times \vec{B}) \tag{5}$$

in a plane perpendicular to $\vec{B}$ and perpendicular to the instantaneous velocity $\vec{v}$. The condition $\vec{F} \perp \vec{v}$ implies that the modulus of the velocity is kept constant, v_0. Let us decompose the vector $\vec{v}$ into a component perpendicular to $\vec{B}$, $\vec{v}_\perp$, and one parallel to $\vec{B}$, $\vec{v}_\parallel$. The force can be expressed as

$$\vec{F} = -e(\vec{v}_\perp \times \vec{B}) \tag{6}$$

and it leads to a circular motion of velocity $|\vec{v}_{0\perp}|$ in the plane perpendicular to $\vec{B}$ and a uniform motion of velocity $|\vec{v}_{0\parallel}|$ along the direction of $\vec{B}$. Therefore, the electron path is a spiral along the vector $\vec{B}$.

If the electron velocity $\vec{v}_0$ is perpendicular to $\vec{B}$, the electron motion is circular; Newton's second law applied to the electron motion reads

$$ev_0 B = m(v_0^2/r), \tag{7}$$

where r is the circle radius and v_0^2/r is the centripetal acceleration. Combining Eqs. (4) and (7) yields

$$\frac{e}{m} = \frac{2U}{B^2 r^2} \tag{8}$$

showing that the electron charge to mass quotient can be calculated from the measurement of the acceleration voltage, magnetic induction and the radius of the circle.

The uniform magnetic field is produced in the experiment by a pair of coils in the Helmholtz arrangement: the two coils have the same radius, their centres lie on the common axis at a spacing equal to their radii and the same current flows through the coils. The magnetic induction B inside such a Helmholtz coil system can be calculated from the average coil radius R, the number of turns n of a coil and the current I in the coils,

$$B = \left(\frac{4}{5}\right)^{3/2} \mu_0 \frac{nI}{R}, \tag{9}$$

where $\mu_0 \approx 1.256\,637\,062\,12(19) \times 10^{-6}\,\mathrm{N{\cdot}A^{-2}}$ is the vacuum magnetic permeability.

Inserting Eq. (9) into Eq. (8), one obtains the final form

$$\frac{e}{m} = \frac{125}{32} \frac{R^2}{\mu_0^2 n^2} \frac{U}{r^2 I^2}. \tag{10}$$

Fig. 1 Experimental set-up for determining the electron charge to mass quotient.

4 Experimental set-up

The experimental set-up is shown in Fig. 1; it contains the following parts:

- Narrow electron beam tube (Fig. 2)

  An electron gun (cathode, control grid and anode, see Fig. 3) inside the tube generates a stream of electrons through thermionic emission at the heated cathode. The electrons are focused in a thin beam by the control grid and accelerated by an electric field between cathode and anode, before they leave upward the electron gun through a small hole in the anode.

  The tube is filled with neon at a low pressure of 4×10^{-6} bar. The electron beam stimulates neon atoms to the emission of light (red), whereby the beam becomes visible within the tube. Graduations inside the tube, via a stepped ladder, permit parallax-free adjustment of the circle diameter. The distance between two steps is of 2 cm;

- Pair of Helmholtz coils of 400 mm diametre ($R = 200$ mm) and number of turns in each coil $n = 154$ connected in series, the polarity being chosen so that the fields of the coils add;

- Power supply for the narrow electron beam tube (Fig. 4): the AC cathode filament voltage (6.3 V/2 A), the DC voltage between control grid and cathode (0 . . . 50 V) and the DC voltage between anode and cathode (0 . . . 300 V).

- DC power supply (0 . . . 18 V, 0 . . . 5 A) for Helmholtz coils (Fig. 5);

Fig. 2 The narrow electron beam tube.

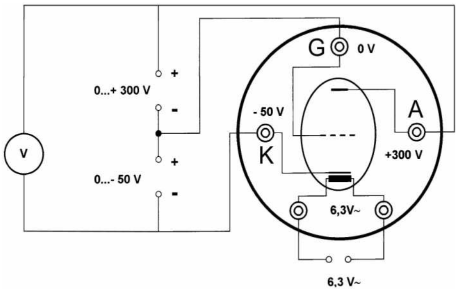

Fig. 3 Electrical circuit of the electron gun.

- Multimetre for measuring the accelerating voltage;
- Multimetre for measuring the current through the Helmholtz coils.

Fig. 4 Power supply for the narrow electron beam tube.

Fig. 5 Power supply for Helmholtz coils.

5 Experimental data and data processing

The effective calculation of the electron charge to mass quotient will be performed using the dependence I^2 versus U/r^2 [see Eq. (10)]:

$$I^2 = \frac{125}{32}\frac{R^2}{\mu_0^2 n^2 (e/m)}\frac{U}{r^2}. \tag{11}$$

I^2 versus U/r^2 is a straight line passing through the origin. The slope and the standard deviation of the slope are calculated based on the least squares method. Then, the calculation of the estimated value of e/m and its standard deviation is straightforward.

Table 1 Experimental data for the electron charge to mass quotient determination. The measurements are performed for a radius $r = 5\,\mathrm{cm}$ of the electron circular trajectory and voltages sufficiently high for which the red light track indicating the electron trajectory is visible.

U/V	165	180	195	210	225	240	255	270	285	300
I/A										

6 Conclusion

Compare your value $-e/m$ with the CODATA value and comment on the difference.