Tutorial 2

1. Dynamics

1.1 A 4 kg object is subjected to two forces: $\vec{F}_1 = 2\vec{u}_x - 3\vec{u}_y$ and $\vec{F}_2 = 4\vec{u}_x - 11\vec{u}_y$ expressed in newtons. The object is at rest at the origin at time $t = 0$. A) What is the object's acceleration? What is its velocity at $t = 3$ s? Where is the object at time $t = 3$ s?

1.2 Find the magnitude and direction of the force acting on the particle of mass m during its motion in the Oxy plane according to the laws of motion: $x = a\sin\omega t$ and $y = b\cos\omega t$, where a, b and ω are constants.

1.3 A body of mass m is thrown at an angle α to the horizontal with an initial velocity $\vec{v}_0$. Find the momentum increment $\Delta\vec{p}$ that the body acquires over the first t second of motion. By considering that the body is thrown from the ground level, find the modulus of the momentum increment Δp during the total time of motion.

1.4 A motor boat of mass m moves on the surface of a lake at a speed v_0. At the moment $t = 0$ the engine is shut down. Assuming the resistance of water to be proportional to the speed of the boat, $F = -rv$, find a) the time interval after which the boat stops; b) the speed of the boat as a function of the distance covered with the shutdown engine.

1.5 An aerostat of mass m starts coming down with a constant acceleration w. Determine the ballast mass to be dumped for the aerostat to reach the upward acceleration of the same magnitude. The air drag is to he neglected.

1.6 At the moment $t = 0$ the force $F = ct$ is applied to a small body of mass m resting on a smooth horizontal plane (c is a constant). The permanent direction of this force forms an angle α with the horizontal . Find: a) Find the dimension of the c constant b) the velocity of the body at the moment of its breaking off the plane (use dimensional analysis to check the plausibility of your result); c) the distance traversed by the body up to this moment (use dimensional analysis to check the plausibility of your result).

1.7 A body which falls freely in air experiences a drag force proportional to its speed $F(v) = -\gamma v$, where γ is an empirical constant which depends on its geometry and on the the air viscozity. a) Knowing that the initial speed of the body is zero, find the time dependence of its speed, $v = v(t)$. b) What is the limit speed? What is the time at which the body reaches 99% of this limit speed?

1.8 A particle of mass m initially at rest moves under the action of a periodic force $F = F_0\sin\omega t$, where F_0 and ω are positive constants. Find the distance covered by the particle in the time interval t.

2. Work. Laws of conservation

(1.119)2.1 A car of mass m starts moving in Ox direction at a speed which depends on the position x according to the law $v = c\sqrt{x}$, where c is a constant. Find the work done by

the resultant external force on the car, during the first t seconds after the beginning of motion.

2.2 A spring with an elastic constant of k hangs vertically. A block of mass m is attached to the unstressed spring and allowed to fall from rest. Find an expression for the maximum distance the block falls before it starts to move upward.

2.3 Standing near the edge of a 20 m high building, you kick a ball with an initial speed of $v_i = 16$ m/s at an angle of 60° above the horizontal. Neglecting air resistance, find a) how high above the building the ball rises, and b) the speed just before it hits the ground.

(1.166) 2.4 A particle of mass 1.0 g moving with velocity $\vec{v}_1 = 3.0\vec{u}_x - 2.0\vec{u}_y$ experiences a perfectly inelastic collision with another particle of mass 2.0 g and velocity $\vec{v}_2 = 4.0\vec{u}_y - 6.0\vec{u}_z$. Find the velocity of the formed particle (both the vector and its modulus). The components of the vectors $\vec{v}_1$ and $\vec{v}_2$ are expressed in the SI units.

(1.168) 2.5 A particle of mass m_1 experiences a perfectly elastic collision with a particle of mass m_2 at rest. What fraction of the incident energy does the incident particle lose if a) it recoils at right angle to the original direction and b) the collision is head-on?