Tutorial 1

1. Significant figures. Accuracy versus precision. Scientific notation

Rules

1) ALL non-zero numbers (1,2,3,4,5,6,7,8,9) are ALWAYS significant.
2) ALL zeroes between non-zero numbers are ALWAYS significant.
3) ALL zeroes which are to the right of the decimal point in a number which is higher or equal to one, including the end of the number are ALWAYS significant.
4) ALL zeroes which are to the left of a written decimal point and are in a number higher or equal to 10 are ALWAYS significant.
5) When a number represents the result of an experimental measurement, the size of the determined uncertainty tells us the the number of significant *digits*. Example: 1.56375 mm is the result of the experimental determination of the fringe spacing in an interference pattern. The uncertaintainty of this measurement was 0.3 mm. This means that the digits 6, 3,7 and 5 are superfluous because they are smaller than the uncertainty. To write the experimental result correctly, first discard all insignificant digits except the leading one, then round off this uncertain digit. In our example, we keep 1.56, then round off to 1.6.

1.1 How many significant figures does each of the following numbers have: a) 145.37; b) 137.089; c) 20.00; d) 20; e) 5600; f)1.34×10^6; g) 0.000402?
1.2 A friend asks to borrow your precious diamond for a day. You are a bit worried, so you carefully have your diamond weighed on a scale which reads 8.17 g. The scale's accuracy is claimed to be ±0.05 g. The next day you weigh the returned diamond again, getting 8.09 g. Is this your diamond?
1.3 Add $\left(9.2\times10^3\text{s}\right)+\left(8.3\times10^4\text{s}\right)+\left(0.008\times10^6\text{s}\right)$. (Hint: When adding, keep the least accurate value.)
1.4 Multiply 2.079×10^2 m by 0.0082 s, taking into account significant figures. (Hint: When multiplying, the result should have as many digits as the number with the least number of significant digits used in the calculation.)
1.5 What is the area and its approximate uncertainty of a circle of radius ($3.8\times10^4\pm0.1$)cm?

2. Dimensions. Dimensional analysis.

2.1 Find the dimensions of some commonly encountered physical quantities in the $LMT\theta I$-class: a) $[v]$ - velocity ; b) $[a]$- acceleration; c)$[F]$- force; d) $[\rho]$ - mass density; e) $[p]$- pressure; f) $[\alpha]$ - angle; g) $[E]$- energy; h) $[S]$- entropy; i) $[Q]$- electric charge; j) $[E]$- electric field intensity; k) $[B]$- magnetic field induction; l) electric permittivitty ε ; m) magnetic permeability.
2.2 It is reasonable to believe that the motion of a mass pendulum depends on its mass m, length l and on the gravitational acceleration g. Use dimensional analysis to find an equation for the period of oscillation of the mass pendulum.
2.3 In an experiment of fluid dynamics it is found that the dynamic pressure of a fluid depends on its density ρ and on the speed v. Use the equation of homogeneity to find an expression for the functional dependence $p=p(\rho,v)$.

2.4 The speed of the electromagnetic waves in vacuum depends on the electric properties of empty space (ε_0) and its magnetic properties (μ_0). Use dimensional analysis to find an equation of $v = v(\varepsilon_0, \mu_0)$.

2.5 A star undergoes some mode of oscillation. Use dimensional analysis and find how does the frequency ω of oscillation depend upon the properties of the star? Assume that the relevant parameters are the star mass density ρ, its radius R, and that the gravitational constant G which appears in Newton's law of universal gravitation.

3. Kinematics

3.1 The position x of an experimental rocket moving along a long rail is measured to be $x(t) = \left(5\frac{\text{m}}{\text{s}}\right)t + \left(8\frac{\text{m}}{\text{s}^2}\right)t^2 + \left(4\frac{\text{m}}{\text{s}^3}\right)t^3 - \left(0.25\frac{\text{m}}{\text{s}^4}\right)t^4$ over the first 10 s of its motion, where t is in seconds and x is in meters. Find the velocity and acceleration of the rocket over the first 10 s and display the results graphically.

3.2 If a particle's position is given by $x(t) = 6 - 12t + 4t^2$ (where t is in seconds, and x is in meters), what is it's velocity at $t = 1\text{s}$?

(b) what is it's speed at $t = 1\text{s}$?

(c) Is there ever an instant when the velocity is 0? If so, give the time.

3.3 The acceleration of a particle which moves with rectilinear translation is given by $a = (t-2)\frac{\text{m}}{\text{s}^2}$. At $t = 0$, the displacement and velocity are zero. a) Find the velocity and displacement when $t = 2\text{s}$ and when $t = 4\text{s}$. b) Show sketches of s, v and a for $0 \le t \le 4\text{s}$. c) Find the average values of the velocity and the acceleration in the intervals $0 \le t \le 2\text{s}$ and $2 \le t \le 4\text{s}$, and discuss the results.

3.4 A particle is moving in a plane with velocity given by $v = v_0\vec{i} + (a\omega\cos(\omega t))\vec{j}$, where $\vec{i}$ and $\vec{j}$ are unit vectors along Ox and Oy axes, respectively. If the particle is at the origin at $t = 0$, a) calculate the trajectory of the particle; b) find the distance from the origin at time $\frac{3\pi}{2\omega}$.