Time, space and velocity in SR

Simultaneous events, clock synchronization, time dilation

As time changes in a Lorentz transformation (7') we must reconsider time measurement.

Model of a light watch. Choose an inertial frame (*S*). It is composed by rigid sticks to measure lengths and by clocks placed in every single point to measure time. The medium between them is homogeneous and isotropic. A convenient theoretical watch is made by a source emitting short burst of light which travel between two parallel mirrors at rest. The light source is situated just near one of the mirrors. Successive reflections indicate ticks and tacks. They are separated by the same time intervals because *c* is a constant.

Synchronization of watches in the same IRF (inertial reference frame). Assume a source situated in the origin emits light bursts. They travel as spherical waves and arrive at watches situated on a sphere at the same moment. We synchronize watches accordingly.

All physical quantities measured upon a body in an IF where this system is at rest are called *proper* or *rest* quantities: proper time, proper length, rest mass, rest energy…

Time dilation. This procedure does not ensure at all synchronization of watches in movement with respect to (*S*). On the contrary, (7') tells another story.

Let's study two events *A* and *B* situated in the same spatial point $x_A = x_B$, $y_A = y_B$, $z_A = z_B$, at two different times $t_B > t_A$. The proper time between these events is $\tau = t_B - t_A = \Delta t > 0$. In the IRF (*S*') moving with constant velocity *V* along the common axis *xx'* the time interval is

$$\Delta t' = t'_B - t'_A = \gamma(t_B - t_A) = \gamma\tau > \tau \qquad (8)$$

Time is dilated when measured from a moving system. Example: decay of unstable particles.

Temporal order of events. What if the two events happen in two different places? Imagine that the event *A* given by (x_A, t_A) is the cause of the event *B* given by (x_B, t_B); therefore $t_B > t_A$. Assume $x_B > x_A$ and represent the propagation velocity of the interaction by $u = \dfrac{x_B - x_A}{t_B - t_A}$. Could the temporal order of the two events be altered by a Lorentz transformation? Computation shows that it is not the case:

$$\left(t'_B - t'_A\right) = \gamma\left(t_B - \frac{V}{c^2} x_B - t_A + \frac{V}{c^2} x_A \right) = \gamma\left(t_B - t_A\right)\left(1 - \frac{uV}{c^2}\right) \qquad (9)$$

The order in time is preserved if $V < c$ and $u < c$. Therefore **the causality is not altered by a Lorentz transformation if reference frames can not move and interactions may not propagates with velocities larger than *c***. We shall assume this is the case for all transformations with physical meaning. Hence **speed of light in vacuum is the maximum velocity for physical interactions.**

Length contraction. The proper length of a rod in the IFR (*S*) is $l_0 = x_B - x_A$. In (S') the positions of the two extremities of the stick must be measured in the same moment $t'_B = t'_A$. Hence:

$$x_B - x_A = \gamma(x'_B - x'_A + \frac{V}{c_2} t'_B - \frac{V}{c_2} t'_A) = \gamma(x'_B - x'_A)$$

$$l' = \gamma l_0 = l_0 \sqrt{1 - \beta^2} \qquad (10)$$

Application: explanation of the negative result of the Michelson experiment.

Velocity composition. In SR the composition of velocities is different from the situation in pre-relativistic physics. The new law must conserve the speed of light and it must reduce to the usual Galilean law for small velocities. To deduce the relativistic relation for velocity composition we use the normal definition of velocity:

In (*S*) $v_x = \frac{dx}{dt}$, $v_y = \frac{dy}{dt}$, $v_z = \frac{dz}{dt}$

In (*S'*) $v'_x = \frac{dx'}{dt'}$, $v'_y = \frac{dy'}{dt'}$, $v'_z = \frac{dz'}{dt'}$

Using the Lorentz relations one finds:

$$v'_x = \frac{v_x - V}{1 - \frac{v_x V}{c^2}} \quad v'_y = \frac{v_y \sqrt{1 - V^2/c^2}}{1 - \frac{v_x V}{c^2}} \quad v'_z = \frac{v_z \sqrt{1 - V^2/c^2}}{1 - \frac{v_x V}{c^2}} \qquad (11')$$

The inverse relations from (S') to (S) are:

$$v'_x = \frac{v_x + V}{1 + \frac{v_x V}{c^2}} \quad v'_y = \frac{v_y \sqrt{1 - V^2/c^2}}{1 + \frac{v_x V}{c^2}} \quad v'_z = \frac{v_z \sqrt{1 - V^2/c^2}}{1 + \frac{v_x V}{c^2}} \qquad (11'')$$

Exercises:

1. Show that if a particle moves with the velocity *c* its speed measured from a moving system is also *c*.

2. A particle moves with velocity $v_x = 0.9c,\ v_y = v_z = 0$ with respect to (*S*). Compute its velocity in (*S'*) which moves with $V \equiv V_x = 0.5c$. The same problem if $V \equiv V_x = -0.5c$.

Conclusion: The velocities of bodies are always smaller than or at most equal to the velocity of light in vacuum.