Galileo's relativity

The movement of a system is described in a 4-dimensional space $R^3 \times R$, R^3 for space and the last R for time. A point in this 4-dimensional space is referred to as an *event*. We are interested in all the transformations which keep constant time intervals and space distances.

Time may vary at most linearly, because the 1st Newton law must remain valid.

Spatial transformations are translations and rotations (not kinematical rotations, changing in time every instant). They maintain also the inertial character of the frame. As an example rotation around axis *Oz* changes only coordinates *x* and *y*, $x' = x\cos\alpha + y\sin\alpha; y' = -x\sin\alpha + y\cos\alpha$, or in matrix form:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \qquad (1)$$

But the most interesting are kinematical transformations.

The Galilean relativity is based on the following principle: **laws of mechanics are the same in all inertial frames (**moving uniformly one with respect to another; in all such frames the 1st law applies). If we know one IF, all frames moving with a constant velocity with respect to this one are also inertial frames. Indeed, assume (*S*) is an IF. If (*S'*) moves with constant velocity $\vec{V}$ with respect to (*S*), then $\vec{r}\,' = \vec{r} - \vec{V}t$. The velocity transforms from (*S*) to (*S'*) by $\vec{v}\,' = \vec{v} - \vec{V}$, and the acceleration remains the same, hence the 2nd Newton law preserves its form.

Exercise 1: Prove formally the last allegation.

Time could be considered the same in all IF, $t' = t$ (Newton). Transformations from (*S*) to (*S'*) could be written in matrix form:

$$\begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & -v_x \\ 0 & 1 & 0 & -v_y \\ 0 & 0 & 1 & -v_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \qquad (2)$$

In the last quarter of the XIX-th century the success of Maxwell's electromagnetic theory pushed physicists to study the behavior of equations relevant to this chapter under a Galilean transformation. One of these relations is the wave equation.

Exercise 2: Prove that the wave equation changes after a Galilean transformation.

Two ways opened to physicists:

- keep the mechanics and Galilean relations as they are and accept the change of the Maxwell's equations
- preserve equations of electromagnetism and hence change equations of mechanics.

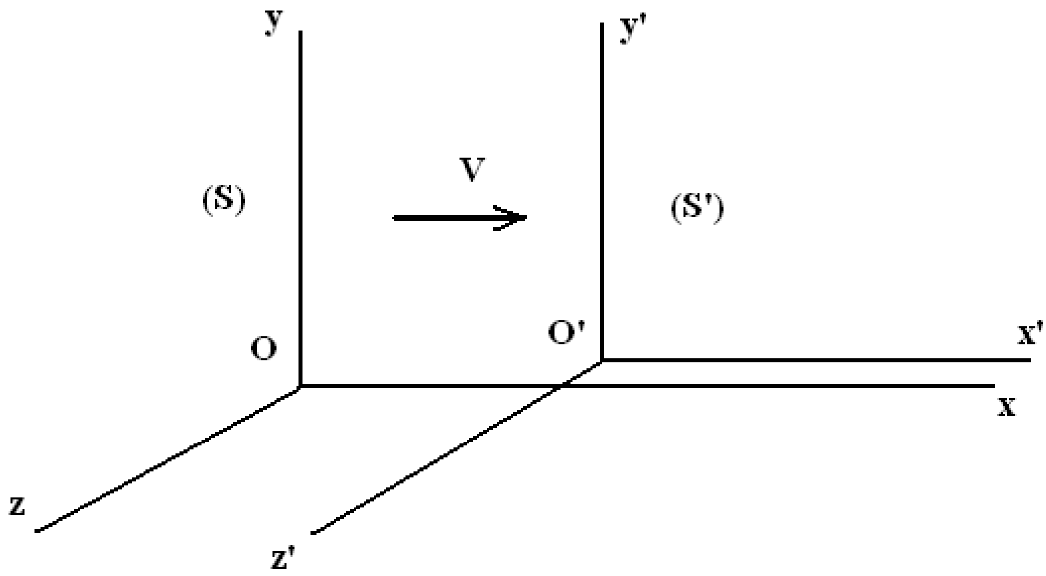
As experiments based on electromagnetism are much more accurate than experiments in mechanics one aimed to find equations maintaining the form of Maxwell's equations. These transformations were found by Lorentz (see below). It appears that Galileo transformations are just a particular case, correct only for small velocities. A consistent theory explaining Lorentz relations was introduced in 1905 by Albert Einstein, the *special relativity.*

Special relativity

Principles of Special Relativity (SR)

1. *The velocity of light in vacuum is a universal constant which do not depend on the inertial frame (IF) from which it is measured.*
2. *The laws of physics are the same in all IF.*

Transformation relations (2) become the *Lorentz transformation*, written below for uniform movement along *xx'* axis.



Assume two IF (S) and (S'). (S') moves with **V** with respect to (S) (assume along the common axes *xx*'). A point has *four* coordinates in each IF: $(x,\ y,\ z,\ t) \in (S)$ or

$(x', y', z', t') \in (S')$. Points in this 4-dimensional space are called *events*. For the convenience put

$$x_1 = x \quad x_2 = y \quad x_3 = z \quad x_4 = ict \tag{3}$$

Allowing for time to vary, transformation writes:

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & \cdots\cdots\cdots & & \alpha_{24} \\ \cdots\cdots\cdots\cdots\cdots & & & \\ \alpha_{41} & & & \alpha_{44} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \tag{4}$$

The α matrix is the Lorentz matrix, having the following form:

$$\alpha = \begin{pmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{pmatrix} \tag{5}$$

where

$$\beta = V/c \qquad \gamma = \left(1-\beta^2\right)^{-1/2} \tag{6}$$

The transformation is:

$$x'_1 = \gamma\left(x_1 + i\beta x_4\right) \qquad x'_2 = x_2 \qquad x'_3 = x_3 \qquad x'_4 = \gamma\left(-i\beta\gamma + x_4\right) \tag{7}$$

Or in normal variables

$$x' = \frac{x - Vt}{\sqrt{1 - V^2/c^2}} \qquad y' = y \qquad z' = z \qquad t' = \frac{t - Vx/c^2}{\sqrt{1 - V^2/c^2}} \tag{7'}$$