Kinematics

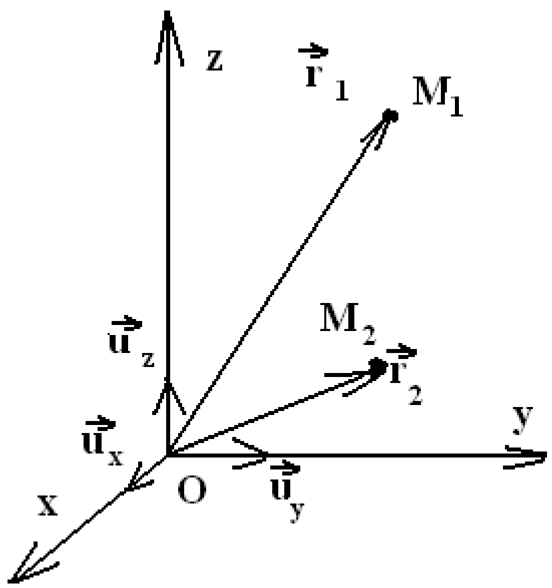
K.1. Definitions and comments

Model: points moving with respect to a **frame of reference**.

Their positions are described by **vectors** in 3D with the origins in a point called **the origin** of the frame. Vectors are right and proper because physics laws expressed in this way have the same form in all frames.

In physics, a point is a mathematical point having a **mass**. The mass is the measure of **inertia (**see the experiment 1 *experienta cu hartia higienica trasa repede si incet*). It does not measure the amount of matter; for this we use the **number of moles**. The mass varies with speed in Special Relativity.

Remark: bodies are formed by many such "physical" points. If the distances between points do not vary the body is rigid. If not we speak about elastic bodies or fluids.



M_1 and *M_2* are two material points. Their positions are given by $\vec{r}_1(t)$ and $\vec{r}_2(t)$, which may vary in time; $\vec{u}_x, \vec{u}_y, \vec{u}_z$ are unit vectors. If $\vec{r}_1$ is a constant vector, point *M_1* is at rest. If $\vec{r}_2(t)$ varies in time point *M_2* moves. Its (instantaneous) **velocity** is defined by:

$$\vec{v}_2(t) = \frac{\mathrm{d}\vec{r}_2}{\mathrm{d}t} \qquad \text{(K1)}$$

Its **speed** is the modulus of the velocity. The average speed (for 1D movement) is the change in the distance divided by the total duration (change in time):

$$v_{av} = \frac{\Delta d}{\Delta t} \qquad \text{(K2)}$$

The variation of the velocity is called (instantaneous) **acceleration**:

$$\vec{a}_2(t) = \frac{\mathrm{d}\vec{v}_2}{\mathrm{d}t} \qquad \text{(K3)}$$

Exercise. Analyze the following affirmations:

- Acceleration and velocity are always parallel
- Acceleration and velocity 1D have always the same direction
- When velocity has a maximum, so does the acceleration
- When acceleration has a maximum, so does the velocity
- When speed is constant acceleration is zero.

K.2. Cartesian coordinates

We use fixed orthogonal axes. Position:

$$\vec{r}(t) = x(t)\vec{u}_x + y(t)\vec{u}_y + z(t)\vec{u}_z \qquad \text{(K4)}$$

Velocity:

$$\vec{v}(t) = \dot{\vec{r}}(t) = \dot{x}(t)\vec{u}_x + \dot{y}(t)\vec{u}_y + \dot{z}(t)\vec{u}_z \qquad \text{(K5)}$$

Acceleration:

$$\vec{a}(t) = \ddot{\vec{r}}(t) = \ddot{x}(t)\vec{u}_x + \ddot{y}(t)\vec{u}_y + \ddot{z}(t)\vec{u}_z \qquad \text{(K6)}$$

Remark: What if the axes move? When we should use such a coordinate system? We have to choose the frame the best suited for the problem, i.e. where the equations are the simplest. One needs a moving frame when one studies the movement of a body on the surface of another body which moves in its turn, as would be a material point moving inside a sphere in rotation. Two obvious frames appear to be interesting: the Earth assumed at rest or the sphere assumed at rest. Other examples are given by problems having certain symmetry: spherical, cylindrical, plane.

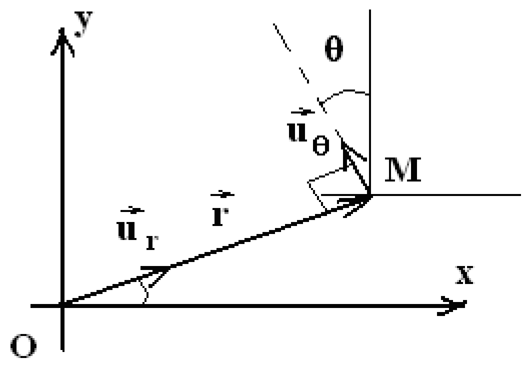
Example.

1.The position of a body is given by $\vec{r}(t) = 6t^2\vec{u}_x - 15\vec{u}_y$. Compute $\vec{v}(t)$and $\vec{a}(t)$. What movement the body has?

Answer: $\vec{v}(t) = 12\,t\,\vec{u}_x - 15\vec{u}_y$; $\vec{a} = 12\vec{u}_x$;

K.3. Polar coordinates

If a body rotates in a plane a polar frame is appropriate.



$$\vec{r} = r\vec{u}_r = x\vec{u}_x + y\vec{u}_y \qquad x = r\cos\theta, \qquad y = r\sin\theta \tag{K7}$$

$$\vec{u}_r = \cos\theta\,\vec{u}_x + \sin\theta\,\vec{u}_y \qquad \vec{u}_\theta = -\sin\theta\,\vec{u}_x + \cos\theta\,\vec{u}_y \tag{K8}$$

Or in matrix form:

$$\begin{pmatrix} \vec{u}_r \\ \vec{u}_\theta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \vec{u}_x \\ \vec{u}_y \end{pmatrix} \tag{K8'}$$

Exercise: express $\vec{u}_x$ and $\vec{u}_y$ in terms of $\vec{u}_r$ and $\vec{u}_\theta$.

If angle θ varies in time, $\theta = \theta(t)$, so do unit vectors $\vec{u}_r$ and $\vec{u}_\theta$.

The velocity and acceleration in polar coordinates are:

$$\vec{v} = \dot{\vec{r}} = \dot{r}\vec{u}_r + r\dot{\vec{u}}_r = \dot{r}\vec{u}_r + r\dot{\theta}\vec{u}_\theta = v_r\vec{u}_r + v_\theta\vec{u}_\theta \tag{K9}$$

$$\vec{a} = \ddot{\vec{r}} = (\ddot{r} - r\dot{\theta}^2)\vec{u}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\vec{u}_\theta = (\ddot{r} - r\dot{\theta}^2)\vec{u}_r + \frac{1}{r}\left[\frac{\mathrm{d}}{\mathrm{d}t}\left(r^2\dot{\theta}\right)\right]\vec{u}_\theta \tag{K10}$$

Particular situation: uniform circular motion, r =const., $\dot{\theta} = \omega$=const.:

$$\vec{v} = r\omega\vec{u}_\theta \qquad \vec{a} = -r\,\omega^2\vec{u}_r\,.$$

Va rog sa insistati mai mult asupra miscarii circulare uniforme, ei nu stiu bine din liceu