D.2. Energetic quantities: kinetic energy, work, total energy

Force is the agent of change, energy is the measure of the change (E. Hecht, Physics, Algebra/Trig, Ch. 6, Energy).

Work is the amount of energy transmitted by a force acting along a certain distance. If the force is constant work is easily defined as follows.

One body. 1D situation: Along a straight line:

$$W = \pm F \cdot x \qquad \text{(D4)}$$

If W>0 work is done by the force, if W<0 work is done against the force.

3D case. Definition for one body:

$$W = \vec{F} \cdot \vec{r} = F\,r\cos\alpha \qquad \text{(D4')}$$

It may be >0, <0, or =0.

For n bodies work is just the sum of expressions of the type (D4) or (D4').

If the force varies one has to consider a small displacement dx and write in 1D case:

$$\delta W = F\mathrm{d}x\,, \qquad \text{hence} \qquad W = \int_{(1)}^{(2)} F\mathrm{d}x \qquad \text{(D5)}$$

In the 3D case however, one may still write the elementary work as

$$\delta W = \vec{F} \cdot \mathrm{d}\vec{r} = F_x\mathrm{d}x + F_y\mathrm{d}y + F_z\mathrm{d}z \qquad \text{(D6)}$$

δW is not the differential of some function, it is just an elementary work, computed for a small distance dx or $\mathrm{d}\vec{r}$.

Functions of several variables !! Partial derivatives !!

And how could we figure out the integral:

$$W = \int_{(1)}^{(2)} \left(F_x \mathrm{d}x + F_y \mathrm{d}y + F_z \mathrm{d}z \right) \qquad ? \qquad \text{(D6')}$$

Line integral or path integral !!

Kinetic energy. By definition kinetic energy of a body of mass *m* moving with velocity $\vec{v}$ is given by:

$$E_k = \frac{m\vec{v}^{\,2}}{2} = \frac{\vec{p}^{\,2}}{2m} \quad \text{(we have used } \vec{p} = m\vec{v}\text{)} \qquad \text{(D7)}$$

Question: what reference frame did we use ?

For *n* material points with masses m_i the definition is:

$$E_k = \sum_{i=1}^{n} \frac{m_i \vec{v}_i^{\,2}}{2} = \sum_{i=1}^{n} \frac{\vec{p}_i^{\,2}}{2m_i} \qquad \text{(D8)}$$

Remark: the factor $1/2$ will become clear later.

Variation of the kinetic energy.

$$\Delta E_k = W_{12} \qquad \text{(D9)}$$

with W_{12} the total work made by *all* forces, internal and external, during the movement from (1) to (2).

Demonstration for 1 body. $E_k = \dfrac{m\vec{v}^{\,2}}{2}$,

$$\mathrm{d}E_k = \vec{v} \cdot m\mathrm{d}\vec{v} = \vec{v} \cdot \mathrm{d}\vec{p} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} \cdot \mathrm{d}\vec{p} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} \cdot \mathrm{d}\vec{r} = \vec{F} \cdot \mathrm{d}\vec{r} = \delta W \qquad \text{(D10)}$$

$$\Delta E_k = \int_1^2 \delta W = W_{12} \,.$$

Try to demonstrate the same relation for n bodies. Faceti va rog demonstartia asta

Remark. In (D9) W_{12} is the work on the path from the point (1) to the point (2). In 2D or 3D **this work depends in general on the actual pathway**. **In the particular case when work does not depend on the actual path,** one says **the force is conservative**, or that **the force is given by a potential. Then and only then the elementary work δW becomes a usual differential.**

Conservative forces. If forces are conservative, $\delta W = -\mathrm{d}U$, with $\mathrm{d}U$ the potential energy of the system. Introducing in (D9):

$$\mathrm{d}E_k + \mathrm{d}U = \mathrm{d}\left(E_k + U\right) = \mathrm{d}E_{tot} = 0, \quad \text{or} \quad E_{tot} = const \tag{D11}$$

Examples: gravitational field, elastic field, but *not* problems with friction.

For the elastic field 1D

$$U = \frac{kx^2}{2} \qquad F = -\frac{dU}{dx} = -kx \tag{D12}$$

In 3D $U = \frac{k\vec{r}^{\,2}}{2}$ and $\vec{F} = -k\vec{r}$.