Bohr model

1.1 Spectral series. The naïve Bohr's theory

Atomic spectra from gases show remarkable regularities. Spectral lines arrange themselves in *series*. A spectral series is a family of lines emitted by an atom whose frequencies may be deduced from a simple law, by the change of just one parameter. The first spectral series was observed by Balmer in 1885, in the visible spectrum of atomic hydrogen. The inverse of the wavelength – (the wavenumber $\tilde{\nu}$ – is given by:

$$\tilde{\nu} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \qquad \text{n=3,4,...} \qquad \text{(Q1.17)}$$

Here $R_H \cong 1.097 \cdot 10^7$ m^{-1} is the Rydberg constant (or $R = 2.07 \times 10^{16}$ rad/s). Other series of atomic hydrogen are obtained from

$$\tilde{\nu} = R_H\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \qquad n>m \qquad \text{(Q1.18)}$$

Similar relations, although more complex, exist for all elements. The *combination principle* (Rydberg-Ritz, 1908) follows: *the wavenumber $\tilde{\nu}_{mn}$ of a spectral line is always given by the difference of two terms, known as spectral terms*:

$$\tilde{\nu}_{mn} = T(m) - T(n) \qquad \text{(Q1.18')}$$

Classical explanations fail to explain such a simple relation.

In 1913 Niels Bohr put forward a new theory based on two postulates and a quantified condition.

1st postulate. Each atom (in general each microsystem) is characterized by a family of stationary states with discrete energies E_1, E_2, E_3,... E_n. When in these states the system does not emit or absorb energy.

2nd postulate. The energy of a microsystem may vary only by discrete values by transitions from a stationary state of energy E_m to a stationary state of energy E_n. The transition is associated to an emission or an absorption of a photon with frequency given by

$$\hbar\omega_{mn} = E_m - E_n \qquad \text{(Q1.19)}$$

If $\omega_{mn} > 0$ the photon is emitted if $\omega_{mn} < 0$ the photon is absorbed.

The Bohr condition (Q1.19) represents the energy conservation for each process of exchange of energy with emission or absorption of photons. It explains the combination principle (Q1.18').

The quantum condition specifies that the angular momentum of an electron in an atom is quantified:

$$l_n = m_0 vr = n\hbar \qquad n=1, 2, 3,\ldots \qquad \text{(Q1.20)}$$

This condition could be understood using a model where the electron moves on a circular orbit around the nucleus as in Fig. 3.
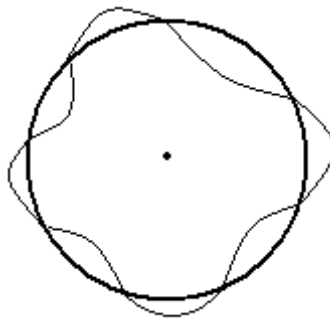

Fig. 3. The orbit of an electron and the de Broglie associated wave.

The electron has an associated wave and as its movement ought to be stable the orbit must contain an integer number of wavelengths: $2\pi r = n\lambda = n\dfrac{h}{p}$. Hence $pr = l = n\hbar$.

Assume a hydrogen atom with the nucleus made of one proton. This proton is more than 1800 times heavier than the electron and we shall consider it at rest. Write the equality of Coulomb and centrifugal forces: $\dfrac{m_0 v^2}{r} = \dfrac{e^2}{4\pi\varepsilon_0 r^2} = \dfrac{e_0^2}{r^2}$. Using (Q1.20) one finds:

$$v_n = \frac{e_0^2}{n\hbar} = \frac{v_1}{n} \qquad\qquad r_n = n^2 \frac{\hbar^2}{m_0 e_0^2} = n^2 r_1 \qquad\qquad n=1, 2, 3,\ldots \qquad \text{(Q1.21)}$$

$r_1 \cong 5.29 \cdot 10^{-11}$ m is the first Bohr radius. Velocities and radii are quantified.

The energy of the electron on the *n*-th orbit is quantified too and given by:

$$E_n = \frac{m_0 v_n^2}{2} - \frac{e_0^2}{r_n} = -\frac{e_0^2}{2r_n} = -\frac{e_0^2}{2r_1}\frac{1}{n^2} = -\frac{1}{n^2}\frac{m_0 e_0^4}{2\hbar^2} = -\frac{13.6}{n^2}\ \text{eV} \qquad \text{(Q1.22)}$$

Exercise: Compute the Rydberg constant as:

$$R_H = \frac{e_0^2}{4\pi c \hbar r_1} = \frac{m_0 e_0^4}{4\pi c \hbar^3} \qquad\qquad \text{(Q1.23)}$$

The existence of stationary states was demonstrated by the Franck-Hertz experiment (http://en.wikipedia.org/wiki/Franck%E2%80%93Hertz_experiment).