Dimensional analysis

All physical quantities have dimensions and measurement units (MU); exception e.g. angles.

1. All physical relations are dimensionally homogeneous.

Examples:

velocity + acceleration = force WRONG

2 m + 3 m = 5 WRONG

3 m/s + 4 m/s = 7 m/s CORRECT

2. The arguments of physical functions have no physical dimensions.

Example:

In $\cos(\omega t - kx)$, if ω is a frequency (measured in s^{-1}) and k is measured in m^{-1} the relation is CORECT. Otherwise it's WRONG

3. A scalar equals a scalar; a vector equals a vector, etc.

4. In mechanics we need 3 fundamental MU: meter for length, kilogram for mass, second for time.

5. Problems.

5.1. Find the velocity of elastic waves in a solid having the density ρ and the Young modulus E.

$[\rho] = ML^{-3}$ $[E] = [F]/[S] = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$ $[v] = LT^{-1}$. The only way to combine density and Young modulus to find a velocity is $v = const\sqrt{E/\rho}$, because $ML^{-1}T^{-2} \times \frac{1}{(ML^{-3})} = L^2T^{-2}$. The standard way to deduce this result follows the procedure from the lecture.

5.2. Other simple examples: kinetic energy, period of an ideal pendulum,…

5.3. Following an explosion in water a gaseous baloon is formed which oscillates with a period T depending of the liquid pressure p, the energy of the explosion ε and the density of the water ρ. Find a correct form of the function $T(p, \varepsilon, \rho)$.

Answer: $T = const \times p^{-5/6} \rho^{1/2} \varepsilon^{1/3}$ (Not all relations are simple !!)