Problems Irodov

6.18. In accordance with classical electrodynamics an electron moving with acceleration w loses its energy due to radiation as $dE/dt = -2(e_0)^2 w^2/(3c^3)$,
where e is the electron charge, c is the velocity of light. Estimate the time during which the energy of an electron performing almost harmonic oscillations with frequency $\omega = 5*10^{15}\ s^{-1}$ will decrease $\eta = 10$ times.

Notation: $\gamma = \frac{2e_0^2}{3c^2}$. If the velocity of the *e* is $v=v_0 \sin\omega t$ the average of the acceleration squared is $\langle a^2 \rangle = \frac{2\pi}{\omega} \int_0^{2\pi/\omega} \left[\frac{d}{dt}(v_0 \sin\omega t) \right]^2 dt = v_0^2 \omega^2 / 2$. The electron energy diminishes in time as

$$\frac{dE}{dt} = -\gamma \langle a^2 \rangle = -\gamma \frac{v_0^2 \omega^2}{2} \times \frac{m}{m} = -\gamma E \frac{\omega^2}{m}$$

$E = E_0 \exp\left[-\gamma \frac{\omega^2}{m} t \right] = E_0 / 10$. Numerical result: $t=1.47 \cdot 10^{-8}$ s.

6.25. Calculate the magnetic field induction at the centre of a hydrogen atom caused by an electron moving along the first Bohr orbit.

The Bohr quantification condition $mv_n r_n = m\omega_n r_n^2 = n\hbar$. The normal acceleration is $a_n = \omega_n^2 r_n = \frac{e_0^2}{r_n^2 m}$. The *e* moving in a circle is equivalent to an electric current $I_n = e\frac{\omega_n}{2\pi}$.

Using Biot-Savart law $B = \mu_0 \frac{I}{2r_n} = 12.5\ \text{T}$

6.31. How many spectral lines are emitted by atomic hydrogen excited to the n-th energy level?

n(*n*-1)/2