# Polarisation by quarterwave plates

## Related topics
Plane, circularly and elliptically polarised light, polariser, analyser, plane of polarisation, double refraction, optic axis, ordinary and extraordinary ray.

## Principle
Monochromatic light falls on a mica plate perpendicular to its optic axis. At the appropriate plate thickness (λ/4, or quarter-wave plate) there is a 90° phase shift between the ordinary and the extraordinary ray when the light emerges from the crystal. The polarisation of the emergent light is investigated at different angles between the optic axis of the λ/4 plate and the direction of polarisation of the incident light.

## Equipment

| | | |
|---|---|---|
| Photoelement f. opt. base plt. | 08734.00 | 1 |
| Lens holder | 08012.00 | 3 |
| Lens, mounted, $f$ = +100 mm | 08021.01 | 1 |
| Diaphragm holder | 08040.00 | 2 |
| Iris diaphragm | 08045.00 | 1 |
| Double condenser, $f$ = 60 mm | 08137.00 | 1 |
| Lamp, f. 50 W Hg high press. lamp | 08144.00 | 1 |
| Power supply for Hg CS/50 W lamp | 13661.97 | 1 |
| Interference filter, yellow, 578 nm | 08461.01 | 1 |
| Polarising filter, on stem | 08610.00 | 2 |
| Optical profile-bench, $l$ = 1000 mm | 08282.00 | 1 |
| Base f. opt. profile-bench, adjust. | 08284.00 | 2 |
| Slide mount f. opt. pr.-bench, $h$ = 30 mm | 08286.01 | 8 |
| Slide mount f. opt. pr.-bench, $h$ = 80 mm | 08286.02 | 1 |
| Polarization specimen, mica | 08664.00 | 2 |
| Digital multimeter | 07122.00 | 1 |
| Universal measuring amplifier | 13626.93 | 1 |
| Condenser holder | 08015.00 | 1 |
| Connecting cord, $l$ = 750 mm, red | 07362.01 | 1 |
| Connecting cord, $l$ = 750 mm, blue | 07362.04 | 1 |

## Tasks
1. To measure the intensity of plane-polarised light as a function of the position of the analyser.

2. To measure the light intensity behind the analyser as a function of the angle between the optic axis of the λ/4 plate and that of the analyser.

3. To perform experiment 2. with two λ/4 plates one behind the other.

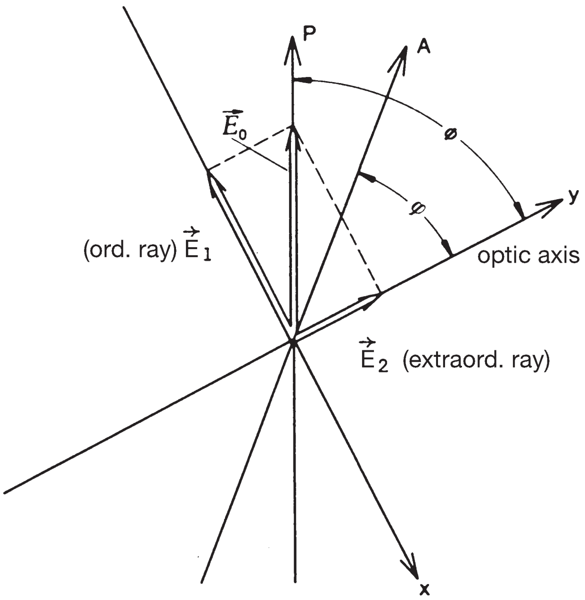
## Set-up and procedure
The experiment is set up as shown in Fig. 1. The experiment lamp with the double condenser (focal length 60 mm) fitted, the lens holder with the iris diaphragm, the lens holder with the interference filter, the polariser, the holder with the λ/4 plate, the lens holder with the lens of focal length 100 mm, the analyser, and the distributor support with the silicon photo-cell are all set up on the optical bench.

First of all the path of the ray is adjusted so that the photo-cell is well illuminated (this is done without the λ/4 plate). With the polariser on zero, the analyser is then rotated until the light which it transmitted is of minimum intensity. The λ/4 plate is now clamped in the holder and rotated so that the light passing through the analyser is again at minimum intensity. The plane of polarisation of the light emerging from the polariser now makes an angle of 0° (or 90°) with the optic axis of the λ/4 plate. The light intensity is measured as a function of the position of the analyser, for angles of 0, 30, 45, 60 and 90°, over the range −90° to +90°. The resistor is plugged in parallel to the entry of the measuring amplifier.

The current intensity of the photo-cell is proportional to the intensity of the incident light.

Fig. 1: Experimental set-up for determining the type of polarisation of the emergent light.

Fig. 2: Splitting of polarised light in a double-refracting crystal (P = polariser, A = analyser).




**Theory and evaluation**

The velocity of the light travelling in the direction of the optic axis of a double-refracting crystal has the same value, $c_o$, whatever the direction of its plane of polarisation. When travelling at right angles to the optic axis, polarised light has the same velocity $c_o$, when the electric vector is perpendicular to the optic axis (ordinary ray, see Fig. 2). If the electric vector is parallel to the optic axis the light velocity $c \neq c_o$ (extraordinary ray).

$E_0$ is the amplitude of an electric field vector emerging from the polariser and $\phi$ the angle between the direction of polarisation $P$ and the optic axis of a double-refracting crystal.

From Fig. 2 we derive the following for the amplitudes of the ordinary and of the extraordinary ray:

$$E_1(t) = E_0(t) \cdot \sin\phi \qquad E_2(t) = E_0(t) \cdot \cos\phi \qquad (1)$$

At time $t$, the state of vibration in the two rays at the crystal surface is described by:

$$E_1(t) = E_0(t) \cdot \sin\phi \cdot \sin\omega t \qquad E_2(t) = E_0(t) \cdot \cos\phi \cdot \sin\omega t \qquad (2)$$

In the case of double-refracting crystals ($\lambda/4$ plates), the thickness

$$d_{\lambda 4} = \frac{\lambda}{4} \cdot \frac{1}{n_o - n_{ao}}, \qquad (3)$$

where $n_o$ is the refractive index of the ordinary ray and $n_{ao}$ that of the extraordinary ray in the crystal, causes a path difference of $\lambda/4$ (i.e. a phase difference of $\pi/2$) between the two rays when they combine to a resultant ray on emerging from the crystal. From (2) we obtain

$$E_x = E_1 = E_0 \cdot \sin\phi \cdot \sin\omega t \qquad E_y = E_2 = E_0 \cdot \cos\phi \cdot \cos\omega t \qquad (4)$$

(4) is the parametric representation of an $E$ vector rotating in the direction of propagation, i.e. perpendicular to the $x$ and $y$ axis, about a fixed axis.

For angles of $\phi = 0°$ and $\phi = 90°$ we obtain plane polarised light of intensity

$$I = I_0 \sim E_0^2 \,. \qquad (5)$$

For an angle of 45°, $\sin\phi = \cos\phi = \frac{1}{\sqrt{2}}$, and the amount of the rotating $E$ vector is

$$E = \sqrt{E_x^2 + E_y^2} = \frac{E_0}{\sqrt{2}} \qquad (6)$$

The light is circularly polarised and of intensity

$$I = \frac{I_0}{2} \sim \frac{E_0^2}{2} \qquad (7)$$

and is transmitted without loss of intensity in all analyser positions.

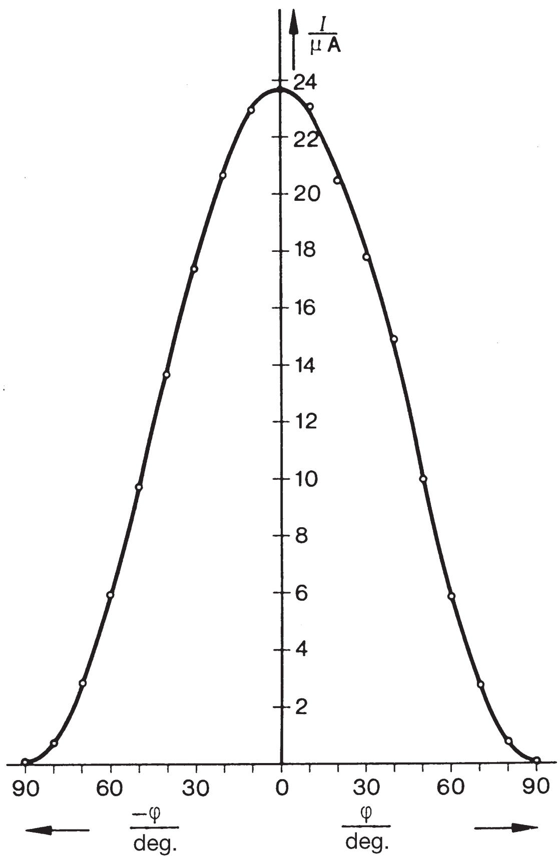
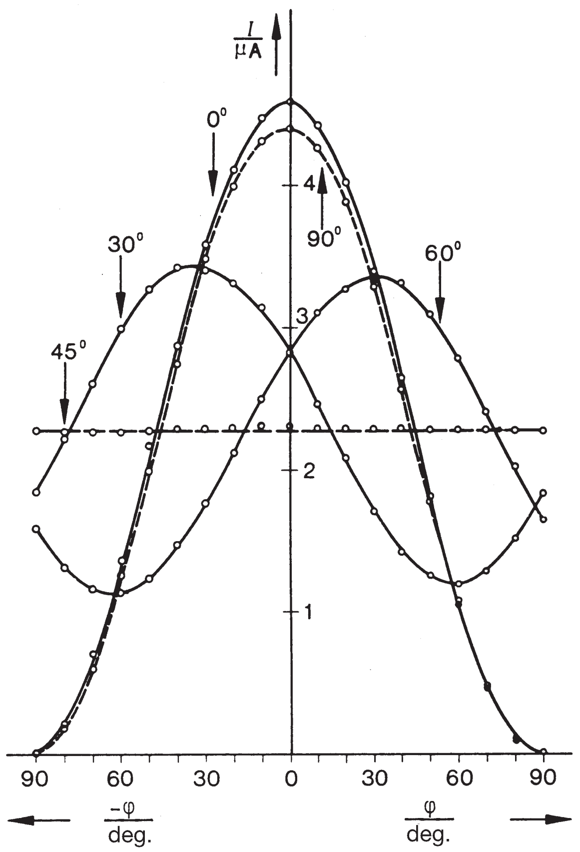



Fig. 3: Intensity distribution of plane-polarised light as a function of the position of the analyser (without $\lambda/4$ plate).

Fig. 4: Intensity distribution of polarised light as a function of the direction of transmission of the analyser: with λ/4 plate at various angular settings.




At all angles $\phi$ other than 0°, 45° and 90°, the transmitted light is elliptically polarised. The tip of the $E$ vector rotating about the axis parallel to the direction of propagation describes an ellipse with the semi-axes.

$$E_a = E_0 \sin \phi \ (x\text{-direction}) \quad (8)$$
$$E_b = E_0 \cos \phi \ (y\text{-direction})$$

For the intensity of the light transmitted by the analyser in the respective directions, we have:

$$I_a \sim E_a^2 = E_0^2 \sin^2 \phi \quad (9)$$
$$I_b \sim E_b^2 = E_0^2 \cos^2 \phi$$

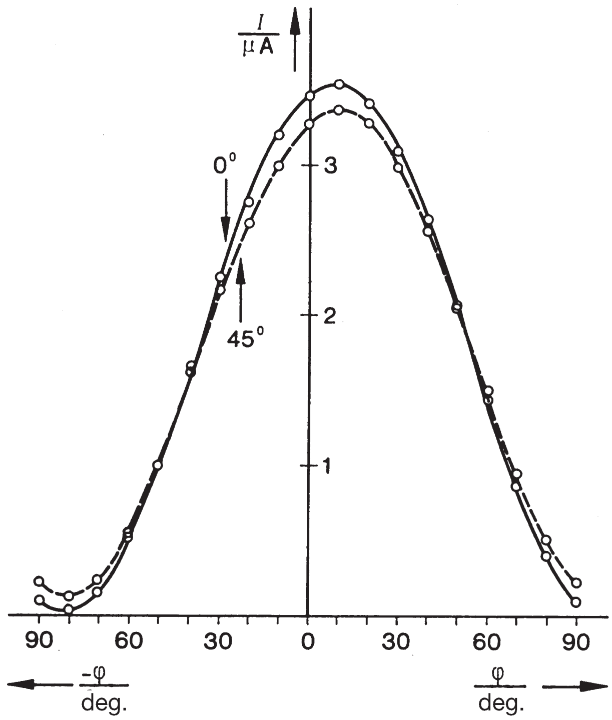
By rotating the analyser we obtain the following for the ratio of the maximum to the minimum transmitted light intensity:

$$\frac{I_a}{I_b} = \frac{E_a^2}{E_b^2} = \frac{\sin^2\phi}{\cos^2\phi} = \tan^2 \phi \quad (10)$$

For any angular setting φ between the analyser and the optic axis of the λ/4 plate, we have:

$$I \sim E_0^2 \cdot \cos^2 \phi \cdot \cos^2 \varphi + E_0^2 \cdot \sin^2 \phi \cdot \sin^2 \varphi \quad (11)$$

Fig. 5: Intensity distribution of polarised light: with λ/2 plate at various angular settings.




First of all the intensity distribution of plane-polarised light is measured as a function of the analyser position, without the λ/4 plate in the path of the rays (Fig. 3).

The type of polarisation of the transmitted light is determined from the corresponding intensity distribution values, for various angles between the optic axis of the λ/4 plate and the direction of transmission of the analyser (Fig. 4).

If two λ/4 plates are set one behind the other, plane-polarised light is produced whatever the direction of the optic axis of the λ/2 plate so created (Fig. 5).