Related topics
Huygens principle, interference, Fraunhofer- and Fresnel diffraction, coherence, laser.

Principle
Multiple slits which all have the same width and the same distance among each other, as well as transmission grids with different grid constants, are submitted to laser light. The corresponding diffraction patterns are measured according to their position and intensity, by means of a photo diode which can be shifted.

Material

1	Laser, Helium-Neon, 1.0 mW, 220 V AC	08181-93
1	Si-Photo detector with amplifier	08735-00
1	Control Unit for Si-Photo detector	08735-99
1	Adapter, BNC-plug/socket 4 mm.	07542-26
1	Optical profile bench, *l* = 1500 mm	08281-00
2	Base for optical profile bench, adjustable	08284-00
5	Slide mount for optical profile bench, *h* = 30 mm	08286-01
1	Slide device, horizontal	08713-00
2	Lens holder	08012-00
1	Object holder, 50 mm x 50 mm	08041-00
1	Lens, mounted, f = + 20 mm	08018-01
1	Lens, mounted, f = + 100 mm	08021-01
1	Diaphragm, 3 single slits	08522-00
1	Diaphragm, 4 multiple slits	08526-00
1	Diffraction grating, 4 lines/mm	08532-00
1	Diffraction grating, 8 lines/mm	08534-00
1	Diffraction grating, 10 lines/mm	08540-00
1	Diffraction grating, 50 lines/mm	08543-00
1	Digital multimeter 2005	07129-00
1	Connecting cord, *l* = 750 mm, red	07362-01
1	Connecting cord, *l* = 750 mm, blue	07362-04

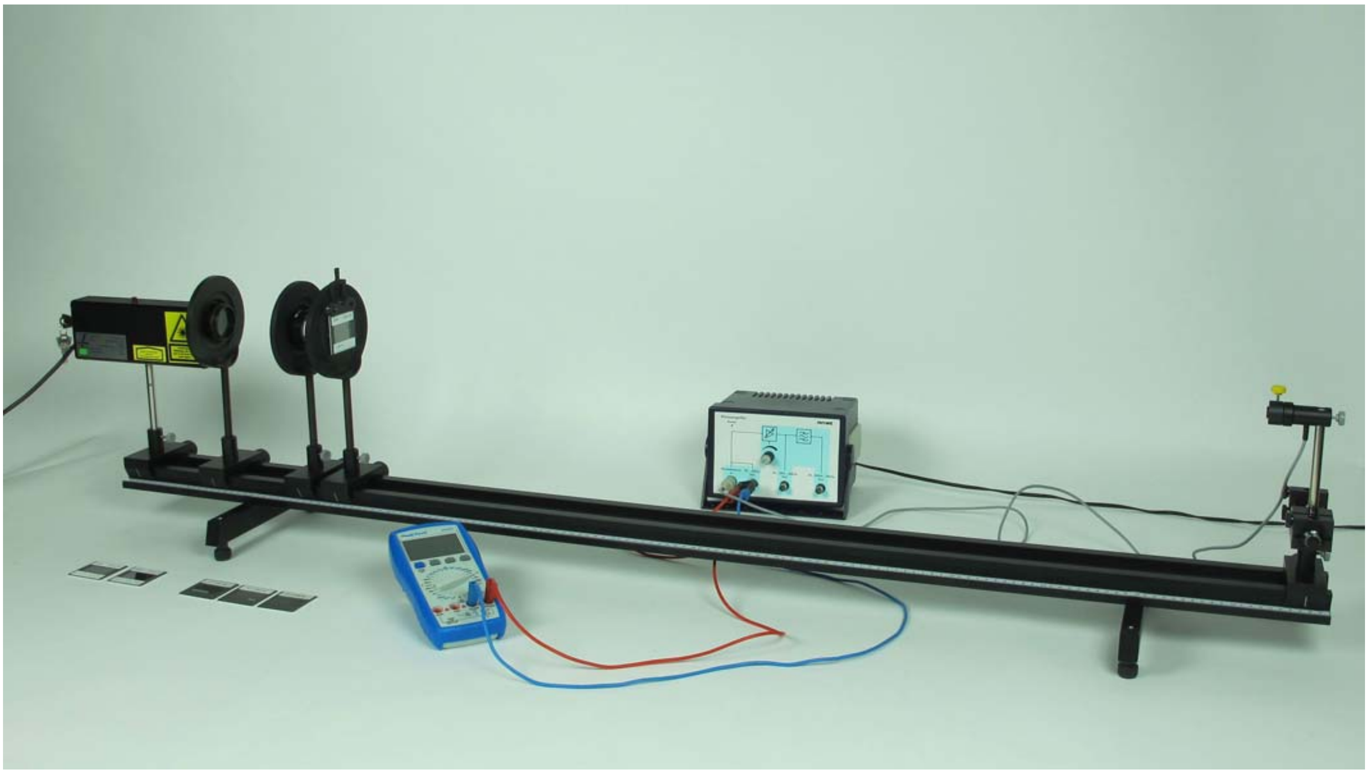

Fig. 1: Experimental set-up to investigate the diffraction intensity of multiple slits and grids. (Positions of the components on the optical bench: laser = 2.5 cm; f/20 mm lens = 14.5 cm; f/100 mm lens = 27.5 mm; diffracting objects = 33 cm; slide mount lateral adjustm., calibr. = 147. 5 cm).

Tasks

1. The position of the first intensity minimum due to a single slit is determined, and the value is used to calculate the width of the slit.
2. The intensity distribution of the diffraction patterns of a threefold, fourfold and even a fivefold slit, where the slits all have the same widths and the same distance among each other, is to be determined. The intensity relations of the central peaks are to be assessed.
3. For transmission grids with different lattice constants *s*, the position of the peaks of several orders of diffraction *k* is to be determined, and the found value used to calculate the wavelength λ of the laser light.

Set-up and procedure

Experimental set-up is shown in Fig. 1. With the assistance of the f = 20 mm and f = 100 mm lenses, a widened and parallel laser beam is generated, which must impinge centrally on the photocell with the slit aperture, the photocell being situated approximately at the center of its shifting range. The diffracting objects are set in the object holder. It must be made sure the diffraction objects which are to be investigated are set vertically in the object holder, and uniformly illuminated.

Caution: Never look directly into a nun attenuated laser beam!
The laser should warm up for about 15 minutes before starting measurements, in order to avoid undesirable intensity fluctuations. The photo detector is connected to the input of the control unit.
The diffraction intensity values are determined for the multiple slits by shifting the photocell in steps of 0.1 mm – 0.2 mm. For the transmission grids, the positions of diffraction peaks must be determined so as to be able to calculate the wavelength of the laser light. For the 50 lines/mm transmission grid, the secondary peaks are outside the shifting range of the photo detector, so that in this case the position of the diffraction reflexes must be marked on a sheet of paper and their distance measured with a ruler.

Theory and evaluation

An optical lattice is a periodic structure consisting of *N* parallel single slits for the diffraction of light. *s* is the lattice constant of the space between slits (measured from center to center) and *b* is the width of a single slit.
Incident light is diffracted by the structures which are in the order of the wavelength λ of the radiation, so that spherical waves are arising.

If a slit width *b* considered against the wavelength is small, so only one elementary wave is transmitted per slit. As the slit width of the grids during this experiment are large compared to the wavelength λ, this approximation can not be used.
The interference pattern of monochromatic light of wavelength λ behind a lattice can be described as a superposition of the Huygens spherical waves of each slit

The path difference d_1 of the marginal rays of a slit of width *b* is

$$d_1 = b \cdot \sin(\varphi)$$

This results in a phase difference of

$$\delta_1 = \frac{2\pi \cdot d_1}{\lambda} = \frac{2 \cdot \pi \cdot b \cdot \sin(\varphi)}{\lambda} \quad (1)$$

The beams of two slits have a path difference of

$$d_2 = s \cdot \sin(\varphi)$$

with *s* as the slit distance (lattice constant).
This results in a phase difference of

$$\delta_2 = \frac{2\pi \cdot d_2}{\lambda} = \frac{2 \cdot \pi \cdot s \cdot \sin(\varphi)}{\lambda} \quad (2)$$

For *N* beams to be deflected to an observation point at the angle of diffraction φ, is obtained with the amplitude E_φ of the diffracted beam and geometric considerations the following dependence of the intensity (the intensity is proportional to the square of the field strength:

$$I_\varphi \propto \frac{\bar{E}_\varphi^2 \cdot \sin^2(N \cdot \delta_2/2)}{\sin^2(\delta_2/2)} \quad (3)$$

In this case, however, E_φ^2 is the intensity of the beam that is diffracted in front of a single slit in the φ direction. Calculations for a single slit follows:

$$\bar{E}_\varphi^2 \propto \frac{\sin^2(\delta_1/2)}{(\delta_1/2)^2} \quad (4)$$

The diffraction intensity of the entire grid is obtained by plugging in equation (4) in eq. (3):

$$I_\varphi \propto \frac{\sin^2\left(\frac{\pi \cdot b}{\lambda} \cdot \sin\varphi\right)}{\left(\frac{\pi \cdot b}{\lambda} \cdot \sin\varphi\right)^2} \cdot \frac{\sin^2\left(\frac{N \cdot \pi}{\lambda} \cdot s \cdot \sin\varphi\right)}{\sin^2\left(\frac{\pi}{\lambda} \cdot s \cdot \sin\varphi\right)} = \frac{\sin^2\left(\frac{\delta_1}{2}\right)}{\left(\frac{\delta_1}{2}\right)} \cdot \frac{\sin^2\left(\frac{N \cdot \delta_2}{2}\right)}{\sin^2\left(\frac{\delta_2}{2}\right)} \quad (5)$$

The first part of the product from(5) is thus the intensity distribution of the single slit, the second part of the result of the interaction of the N slits. Thus it becomes obvious that the minima of the single slits also retained in the grid, because the first factor becomes zero, so the product is also zero.
According to Fraunhofer the minima and maxima of a single slit are referred to as first class interference, while the interactions of several slits leads to second class interference.
The observation of a single slit (first factor) results in an intensity minimum when the numerator from eq. (5) is zero. In this case:

$$\sin\varphi_h = \frac{h \cdot \lambda}{b} \quad (h = 1, 2, 3, \ldots) \quad (6)$$

The angular position of the 1st class peaks is given approximately through:

$$\sin\varphi_h = \frac{2h+1}{2} \cdot \frac{\lambda}{b} \;;\; (h = 1, 2, 3, \ldots) \quad (7)$$

If several slits act together, the minima of the single slits always remain. Supplementary 2nd class minima appear when the 2nd factor also becomes zero.
For a double slit (*N* = 2), the zero points can be easily calculated after application of an

addition theorem to the second factor of equation 5(N = 2) from the following conditional equation:

$$4\cos^2\left(\frac{\pi}{\lambda}\cdot s\sin\varphi\right)=0 \quad (8)$$

This term is zero for

$$\sin\varphi_k=\frac{2k+1}{2}\cdot\frac{\lambda}{s} \;\; ; \;\; (k=0,1,2,3,...) \quad (9)$$

It is easily seen to equation (5), the second term oscillates faster since s>b and therefore $\delta_2>\delta_1$ and also N > 1 (see Fig. 2). For the intensity I of the second-order principal maxima applies in addition to the grid, as the second factor of eq. (5) is apparent:

$$I\propto N^2 \quad (10)$$

The main 2nd class peaks thus become more prominent as the number of slits increases. There still are (N-2) secondary 2nd class peaks between the main peaks.
When light is diffracted through transmission grids with lattice constant s, the diffraction angle φ of the main peaks fulfills the following relation:

$$\sin\varphi_k=\frac{k\cdot\lambda}{s} \quad (k=1,2,3,...) \quad (11)$$

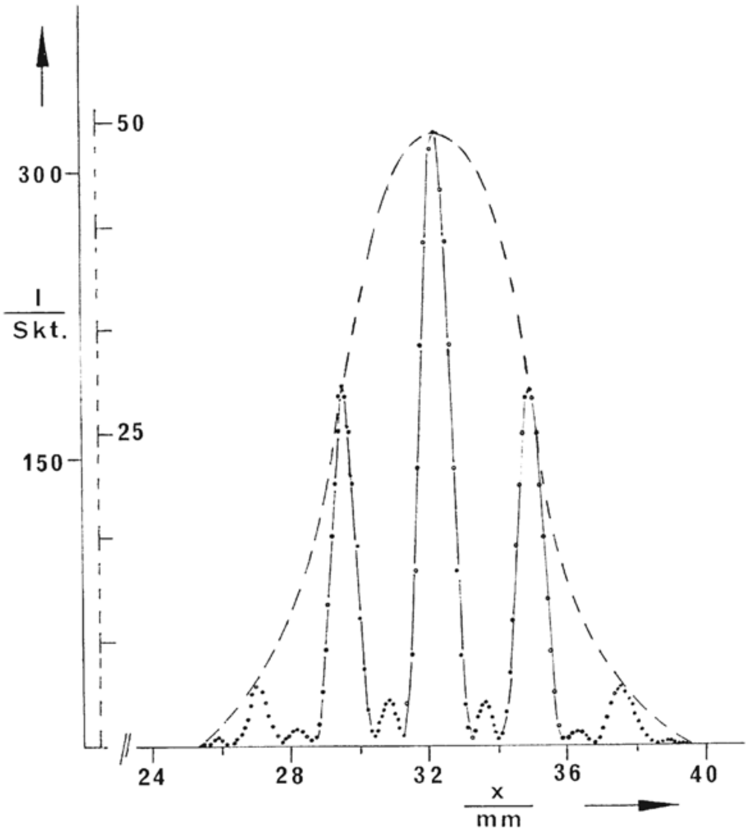

Fig. 2: Diffraction intensity I as a function of the position x for a threefold slit, b_1 = 0.1 mm and s = 0.25 mm. Distance between threefold slit and photo detector: L = 107 cm. For comparison, the intensity distribution of a single slit b = 0.1 mm, is entered as a dotted line.

In Fig. 3 the diffraction intensity I is shown for a threefold slit in dependence on the position x of the photo detector (distance between the slit and photo detector L = 107 cm).

On display are five main peaks (0., ± 1. and ± 2. order) second class with the N-2=1 intermediate secondary maxima.
The slit width *b* is expressed in the envelope which which is adapted in the figure on the ordinate: This represents the maximum 0th order of the first class.
According to equation (5) can be seen that the envelope (ie, the interference 1st class) the interference pattern more strongly attenuates by the lattice periodicity to increasing orders, the wider the single slit *b*. One obtains the width of the slits b_1 = 0.097 mm from (6), with the distance $2\cdot\Delta x=14\,mm$ between the two 1st class minima, so the envelope($\sin\varphi\approx\tan\varphi, L=107\,cm, \lambda=632.8\,nm$).
Fig. 3 shows the diffraction figure of a fourfold slit. In this case, the number of 2nd class peaks is (N - 2) = 2. In the same way, diffraction through a fivefold slit (no figure) yields (N - 2) = 3 2nd class secondary peaks.

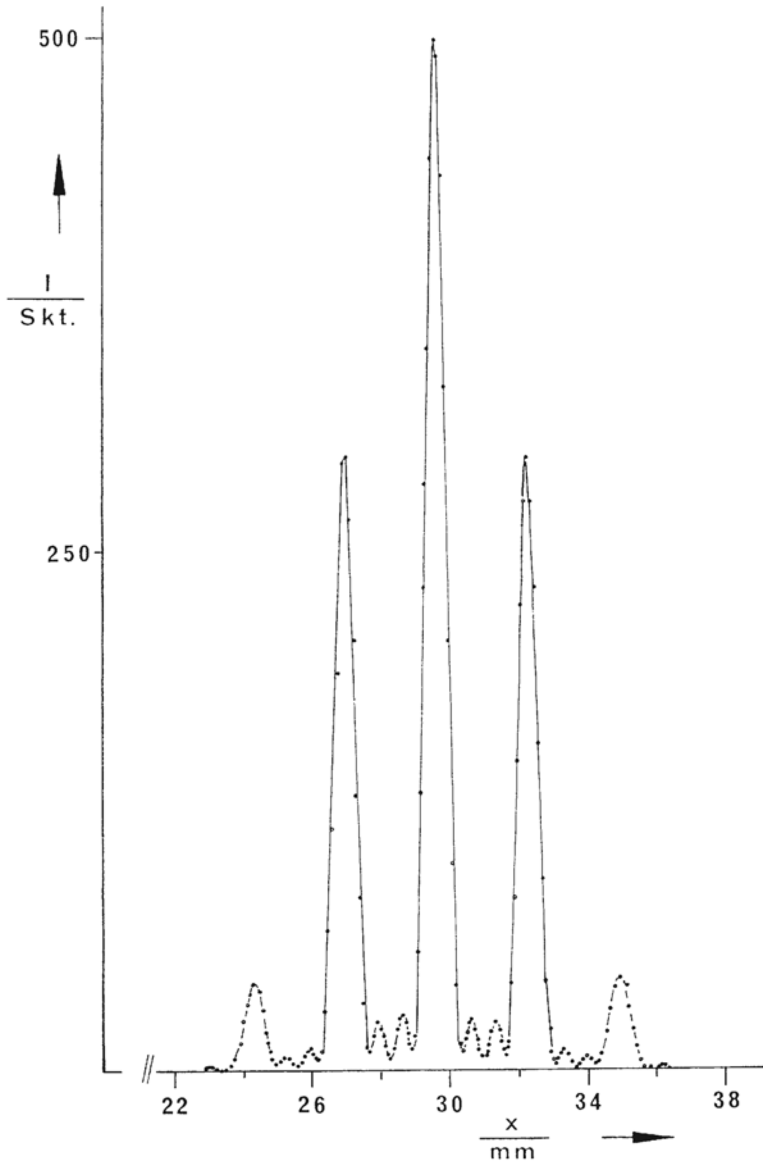

Fig. 3: Diffraction intensity *I* as a function of the position *x* for a fourfold slit with b_1 = 0.1 mm and *s* = 0.25 mm

Table 1 gives the intensity values of the central peaks of the diffracting objects with N = 3 till N = 5, as well as the relative values determined empirically and according to equation (10).

	exp.	theor.
I_{05} (p=5) = 720 Skt.		
I_{04} (p=4) = 500 Skt.	I_{05} / I_{04} = 1.44	$(5/4)^2$ = 1.56
I_{03} (p=3) = 300 Skt.	I_{05} / I_{03} = 2.40	$(5/3)^2$ = 2.78

Table 1:

Fig. 4 shows the distances Δx between the transmission maximum (k = 0) and the diffraction peaks measured for 4 different transmitting grids up to the 3rd order (k = 3) as a function of the lattice constant s. With equation (11) Fig. 4 yields λ = 635 nm as an average value for the wavelength of the used laser light.

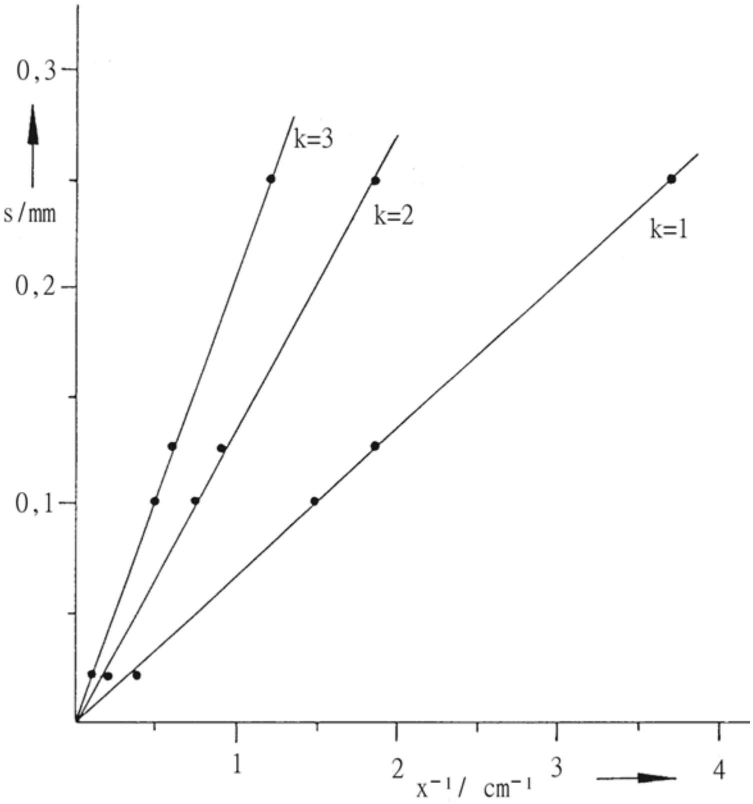

Fig. 4: Reciprocal distance of the diffraction peaks up to the 3rd order of diffraction (k = 3) as a function of the lattice constant.

Plotting the lattice constants of the reciprocal spacing of the diffraction maxima results in straight lines.

