II.3. DETERMINATION OF THE ELECTRON SPECIFIC CHARGE BY MEANS OF THE MAGNETRON METHOD

1. Work purpose

The work purpose is to determine the ratio between the absolute value of the electron charge and its mass, e/m, using a device called magnetron. In this device, the trajectories of the electrons emitted by a heated filament are modified by an externally applied magnetic field.

2. Theory

The method to determine the electron specific charge e/m is based on the study of the electron movement in electric and magnetic fields. The force acting upon a particle of charge $q = -e$ under these fields is called Lorentz force and is given by the formula:

$$\vec{F} = -e(\vec{E} + \vec{v} \wedge \vec{B}), \tag{1}$$

where $\vec{v}$ is the electron velocity, $\vec{E}$ is the electric field intensity, and $\vec{B}$ is the magnetic field induction.

According to Newton's second law:

$$\vec{F} = m\vec{a}, \tag{2}$$

the equation of the electron movement is of the form:

$$\frac{d\vec{v}}{dt} = -\frac{e}{m}(\vec{E} + \vec{v} \wedge \vec{B}). \tag{3}$$

To determine the electron specific charge, we will use the magnetron method. The magnetron is a cylindrically symmetric vacuum diode, placed inside a concentric solenoidal coil. Its section is presented in Figure 1. The cathode C, formed by a wire that also serves as filament, is coaxial with the cylindrical anode A and with the coil S, so that the magnetic field induction vector $\vec{B}$ is paralel with the magnetron symmetry axis.
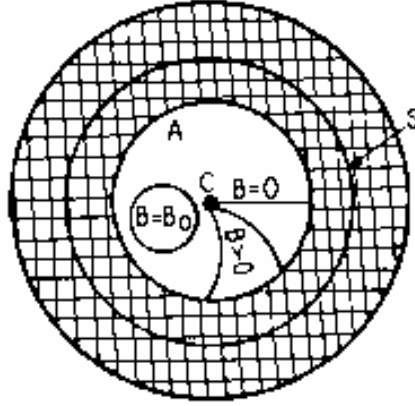
69

Figure 1.

When $B = 0$, the electrons emitted by the cathode through thermoelectronic emission, will move radially towards the anode under the influence of the electric field $\vec{E}$ produced by the bias U applied to the tube.

When $B \neq 0$, the electrons suffer a deviation orthogonal to $\vec{v}$, due to the magnetic field. Their trajectories, starting from the cathode and ending on the anode, curve themselfs. If the magnetic field becomes great enough, then it is possible that the electrons can never reach the anode. This happens when their trajectories become circular, with the radius $r = R/2$. In this situation the electrons form a space charge region around the cathode, screening it, and the anodic current practically drops to zero. We will try to find out a relation that will give us the expression of the electron specific charge e/m, starting from this experimental situation.

Due to the magnetron symmetry, we will use cylindrical coordinates r, θ, z, so that $\vec{E} = -E\,\vec{u}_r$ and $\vec{B} = B\,\vec{u}_z$. The unitary vectors $\vec{u}_r$, $\vec{u}_\theta$, are not constant (see Figure 2), but are given by the relations

$$\vec{u}_r = \cos\theta \cdot \vec{u}_x + \sin\theta \cdot \vec{u}_y, \qquad \vec{u}_\theta = -\sin\theta \cdot \vec{u}_x + \cos\theta \cdot \vec{u}_y, \qquad (4)$$

so that their time derivatives are

$$\dot{\vec{u}} = \dot{\theta}\,\vec{u}_\theta, \qquad \dot{\vec{u}} = -\dot{\theta}\,\vec{u}_r. \qquad (5)$$

The velocity is then

$$\vec{v} = \dot{r}\,\vec{u}_r + r\,\dot{\theta}\,\vec{u}_\theta + \dot{z}\,\vec{u}_z \tag{6}$$

and the vectorial Equation (3) takes the form

$$\ddot{r} - r\,\dot{\theta}^2 = \frac{e}{m}E - \frac{e}{m}r\,\dot{\theta}\,B, \tag{7}$$

$$2\,\dot{r}\,\dot{\theta} + r\,\ddot{\theta} = \frac{e}{m}\dot{r}\,B, \tag{8}$$
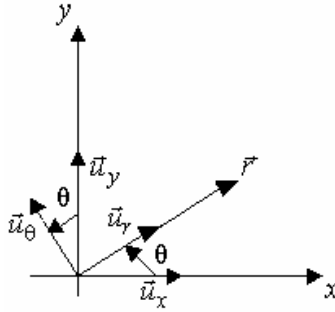
$$\ddot{z} = 0. \tag{9}$$

Figure 2.

To find out the electron trajectory, we will suppose that the time dependence of any quantity is implicit through the time dependence of the radius, that is

$$\frac{d}{dt} = \dot{r}\,\frac{d}{dr}. \tag{10}$$

Then Equation (8) becomes

$$r\,\frac{d\dot{\theta}}{dr} + 2\,\dot{\theta} = \frac{e}{m}B, \tag{11}$$

which has the solution

$$\dot{\theta} = \frac{A}{r^2} + \frac{eB}{2m}. \tag{12}$$

To find out the integration constant A, we will suppose that the electrons are emitted orthogonal to the cathode surface (this assumption is supported by the action of the image force that appears when a charge is close to a metallic surface), so that

$$\vec{v}(R_C) = v_0 \, \vec{u}_r,$$

(13)

R_C being the cathode radius. From this condition we obtain

$$\dot{\theta} = \frac{eB}{2m}\left[1 - \left(\frac{R_C}{r}\right)\right]$$

(14)

and also, from Equation (9),

$$\dot{z} = 0.$$

(15)

As

$$E = \frac{dV}{dr},$$

(16)

where V is the potential, the relation (7) will give

$$\frac{d\dot{r}^2}{dr} = 2\frac{e}{m}\frac{dV}{dr} - \frac{e^2 B^2}{2m^2}r\left[1 - \left(\frac{R_C}{r}\right)^4\right].$$

(17)

Under the conditions (13) and

$$V(R_C) = 0, \qquad V(R_A) = U,$$

(18)

the solution of Equation (17) is

$$\dot{r}^2 = v_0^2 + 2\frac{e}{m}V - \frac{e^2 B^2}{4m^2}r^2\left[1 - \left(\frac{R_C}{r}\right)^2\right]^2.$$

(19)

At the anode radius R_A,

$$\dot{r}^2\Big|_{r=R_A} = v_0^2 + 2\frac{e}{m}U - \frac{e^2 B^2}{4m^2}R_A^2\left[1 - \left(\frac{R_C}{R_A}\right)^2\right]^2.$$

(20)

This means that the only electrons that can reach the anode are those for which the right hand side of Eq. (20) is non-negative. If $v_0 = 0$, this means

$$B^2 \leq B_0^2 = 8\frac{m}{e}\frac{U}{R_A^2}\left[1 - \left(\frac{R_C}{R_A}\right)^2\right]^2 \cong 8\frac{e}{m}\frac{U}{R_A^2}$$

(21)

(as $R_C \ll R_A$). Otherwise, we need

$$v_0^2 \geq \frac{e^2\left(B^2 - B_0^2\right)}{4m^2} R_A^2 \left[1 - \left(\frac{R_C}{R_A}\right)^2\right]^2 = v_{\min}^2 . \tag{22}$$

From equation (21) we also obtain the electron specific charge as

$$\frac{e}{m} = \frac{8U}{R_A^2 B_0^2} . \tag{23}$$

The conduction electrons in metals are described by the Fermi-Dirac statistics. However, as the extraction work is much greater than the thermal energy $k_B T$, the emitted electrons obey a Maxwell-Boltzmann statistics,

$$n(v) \propto v^2 \exp\left[-\frac{mv^2}{2k_B T}\right] . \tag{24}$$

Although this relation is not consistent with the assumption (13), it gives us a good description of the emitted electrons behaviour. As long as $B < B_0$, all the emitted electrons reach the anode, so that

$$i = i_0 = \int_0^\infty eSn(v)v\,dv \propto \int_0^\infty v^3 \exp\left[-\frac{mv^2}{2k_B T}\right] dv . \tag{25}$$

When $B \geq B_0$,

$$i \propto \int_{v_{\min}}^\infty v^3 \exp\left[-\frac{mv}{2k_B T}\right] dv , \tag{26}$$

so that we obtain

$$i = i_0 \left[1 + C\left(B^2 - B_0^2\right)\right] \exp\left[-C\left(B^2 - B_0^2\right)\right], \tag{27}$$

where

$$C = \frac{e^2 R_A^2}{8mk_B T} \left[1 - \left(\frac{R_C}{R_A}\right)^2\right]^2 \cong \frac{e^2 R_A^2}{8mk_B T} . \tag{28}$$

As the magnetic field induction for a coil is

$$B = \mu_0 nI = \mu_0 \frac{N}{l} I, \qquad (29)$$

we can write

$$i = i_0 \left[1 + D\left(I^2 - I_0^2\right)\right]\exp\left[-D\left(I^2 - I_0^2\right)\right], \qquad (30)$$

$$D = \mu_0^2 n^2 C, \qquad (31)$$

$$\frac{e}{m} = \frac{8}{\mu_0^2 n^2 R_A^2} \cdot \frac{U}{I_0^2} = K \frac{U}{I_0^2}. \qquad (32)$$

The logarithm of equation (30) is

$$\log \frac{i_0}{i} = D\left(I^2 - I_0^2\right) - \log\left[1 + D\left(I^2 - I_0^2\right)\right]. \qquad (33)$$

When

$$0 < D\left(I^2 - I_0^2\right) << 1, \qquad (34)$$

this becomes

$$\log \frac{i_0}{i} \cong \frac{D^2}{2}\left(I^2 - I_0^2\right)^2 - \frac{D^3}{3}\left(I^2 - I_0^2\right) + ..., \qquad (35)$$

so that

$$\sqrt{\log \frac{i_0}{i}} \cong \frac{D}{\sqrt{2}}\left(I_0 - I_0^2\right) - \frac{D^2}{3\sqrt{2}}\left(I^2 - I_0^2\right)^2 + ..., \qquad (36)$$

which represents an almost straight line. The corresponding graph is presented in Figure 3. One can see that, using this function the determination of I_0^2 is rather easy.

The value of the constant K is well determined in each experimental case (when n and R_A are known) and must be expressed in I.S., so that, by replacing the voltage U (in volts) and the current I_0 (in amperes), we obtain the specific charge e/m (in C/kg). For the device used in the present work, the value of the constant is $K = 2{,}25 \cdot 10^9$ (I. S.).
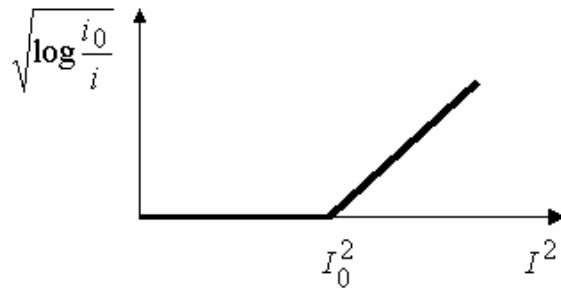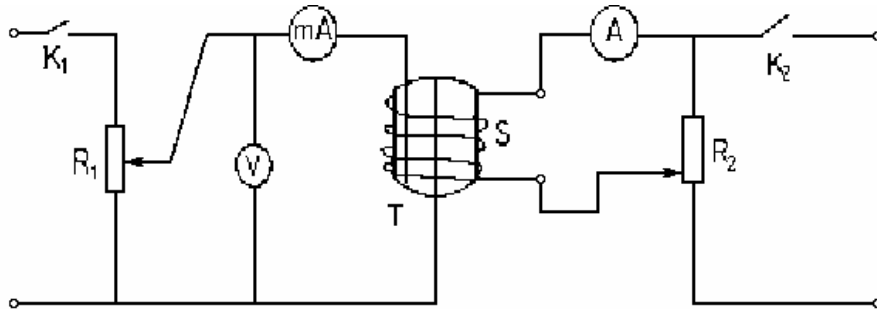
Figure 3.

3. Experimental set-up

Figure 4.

The draft of the experimental set-up, presented in Figure 4, is composed of two circuits : in the left part is the circuit of the tube and in the rigth part the one of the solenoid.This includes:

- The magnetron tube, T.
- The solenoid, S.
- The voltmeter V, to measure the voltage applied to the tube.
- The milliammeter mA, to measure the anodic current; we will use the scale of 0.006 A.
- The ammeter A, to measure the solenoid current; we will use the scale of 0.600 A.
- The rheostats R_1 and R_2 , used as potentiometers.
- The switches K_1 and K_2.

4. Working Procedure

The above set-up is entirely assembled on a work bed and is feeded from the network through a d.c. rectifier. By turning on the switches K1

and K2, both circuits will have a voltage such that with the potentiometer R1 we can vary the voltage U applied to the tube and with the potentiometer R2 we can vary the current I that flows through the solenoid

The measurement of the anodic current i in order to obtain the value I_0 is made by keeping the bias U constant. The variation steps for the current I are chosen such that the readings on the milliammeter scale could be made with the highest possible accuracy (i.e. 25 mA).The measurements must be performed for three different values of the bias, $U_1 < U_2 < U_3$, conveniently chosen. The results will be written down in Table 1.

Table 1

$U_1 =$	$I(A)$	...
30V	$i(A)$	...
$U_2 =$	$I(A)$	...
40V	$i(A)$	...
$U_3 =$	$I(A)$	...
50V	$i(A)$	...

5. Experimental data processing

For the three different values of the voltage, U_1, U_2, U_3, one plots the graphs $\sqrt{\log(i_0/i)} = f(I^2)$ (see Figure 3).

Three distinct values will be obtained in this way for I_0,corresponding to the three values of the bias U, for which e/m is to be computed according to the relation (32). The average of the three obtained values e/m is considered to be the closest result to the real value.

6. Questions

1. How do the electrons move, compared to the direction of the external applied electric field $\vec{E}$?

2. How do the electrons move raported to the direction of the magnetic induction $\vec{B}$?

3. What is the physical meaning of the relation (19)?