

APPENDIX

A. EXPERIMENTAL DATA PROCESSING

I. ELEMENTS OF ERROR CALCULATIONS

Any experimental measurement is affected by errors. Depending on their cause, they can be divided into 3 categories: systematic, random and rough errors.

1. The systematic errors have three possible sources:

a) *Observation errors*. If, for instance, the observer reads the indications of the measurement device while looking at it in an oblique way, all of his readings will be higher or smaller than the real values. These errors can be completely eliminated by correcting the observer's working method.

b) *Device errors*. Any measurement device has a scale (for the digital display devices, we can consider the scale as implicit). No reading made on this scale can be more accurate than half of the smallest scale division. These errors can be reduced (by replacing the used device with a more accurate one), but they cannot be completely eliminated.

c) *Method errors*. During the process of measuring, the system that is measured interacts with the measurement device, and this interaction modifies the results of the measurement. For instance, in order to measure a resistance, we can use the upstream or the downstream methods. In the first case the value obtained is bigger than the real one ($R_{meas} = R(1 + R_A/R)$), and in the second one it is smaller ($R_{meas} = R/(1 + R/R_V)$). We can eliminate these errors if we know the internal resistances of the

measurement devices (which means to measure other resistances), or if we replace this method with a bridge one, which compares the unknown resistance with other ones, assumed as known (this implies, again, measuring other resistances). Therefore, these errors can be reduced, but they cannot be completely eliminated.

Whatever the causes of systematic errors may be, they share one feature: the value of an individual measurement is the same every time we repeat the measuring, therefore the error is also the same. For this reason, the calculation of errors for indirect measurements is done in the same way for all systematic errors.

The absolute error δ_x of a measured quantity x represents the modulus of the maximum possible difference between the measured and the real value. The relative error ε_x is expressed by the ratio between the absolute error and the modulus of the real value (under the condition that the denominator is non-null).

Then, if an indirectly determined value results from the relation

$$z = x \pm y, \quad (1)$$

its absolute error is

$$\delta_z = \delta_x + \delta_y, \quad (2)$$

while if the value results from the relation

$$z = xy^{\pm 1}, \quad (3)$$

its relative error is

$$\varepsilon_z = \varepsilon_x + \varepsilon_y. \quad (4)$$

2. The random errors are due to statistical reasons. Experience has proven that all magnitudes directly measured can be classified in two

possible groups: discrete (for instance, the number of impulses recorded by a detector), and continuous.

The theoretical analysis of the statistics of discrete quantities proves that their values are distributed in agreement with the Poisson probability distribution. According to this, the probability to obtain a number n of impulses for one measurement is

$$p(n) = e^{-a} \frac{a^n}{n!}, \quad (5)$$

where

$$a = \sum_{n=0}^{\infty} np(n) \quad (6)$$

is the “true” value of the number of impulses (and, generally, it is a real number), and the error in the determination of the number a (standard error or mean square deviation) is

$$\sigma_a = \sqrt{\sum_{n=0}^{\infty} (n-a)^2 p(n)} = \sqrt{a}. \quad (7)$$

If we make a number N of measurements in identical circumstances, obtaining the values $n(1)$, $n(2)$, ..., $n(N)$, then the estimate of the true value is given by the mean value

$$\text{Est}a \equiv \tilde{n} = \frac{1}{N} \sum_{i=1}^N n(i). \quad (8)$$

Then, the error affecting an individual measurement will be

$$\sigma_{n(i)} = \sqrt{n(i)} \quad (9)$$

and that of the average value will be

$$\sigma_{\tilde{n}} = \sqrt{\frac{\tilde{n}}{N}}. \quad (10)$$

Let us move on to the case of continuous quantities. Statistical physics prove that the values of these quantities are distributed in agreement with the normal (Gauss) probability distribution. Let us first consider the case of a single quantity x . Then, its probability density will be

$$P(x) \equiv \frac{dp(x, x + dx)}{dx} = \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left[-\frac{(x - a_x)^2}{2\sigma_x^2}\right], \quad (11)$$

where

$$a_x = \int_{-\infty}^{\infty} xP(x)dx \quad (12)$$

is its “true” value and

$$\sigma_x = \sqrt{\int_{-\infty}^{\infty} (x - a_x)^2 P(x)dx} \quad (13)$$

is its standard error. In the case when we make a number N of measurements under identical circumstances and we have as a result the variables $x(1)$, $x(2)$, ..., $x(N)$, then the estimate of the true value is given by the mean value

$$\text{Est}a_x \equiv \tilde{x} = \frac{1}{N} \sum_{i=1}^N x(i). \quad (14)$$

The error affecting an individual measurement $x(i)$ will be

$$\sigma_{x(i)} = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x(i) - \tilde{x})^2} \quad (15)$$

(as we only have $N-1$ independent deviations $x(i) - \tilde{x}$) and that of the mean value will be

$$\sigma_{\tilde{x}} = \frac{\sigma_{x(i)}}{\sqrt{N}}. \quad (16)$$

Let us now consider the case of n quantities x_1, x_2, \dots, x_n , which form a vector in a n -dimensional space. In this case, the normal distribution will be

$$\mathbf{P}(\mathbf{x}) = \frac{1}{\sqrt{(2\pi)^n \det \bar{\Gamma}}} \exp\left[-\frac{1}{2}(\mathbf{x} - \mathbf{a})^T \bar{\Gamma}^{-1}(\mathbf{x} - \mathbf{a})\right], \quad (17)$$

where the covariance matrix $\bar{\Gamma}$ is defined as

$$\Gamma_{i,j} \equiv \rho_{i,j} \sigma_i \sigma_j = \int_{-\infty}^{\infty} (x_i - a_i)(x_j - a_j) \mathbf{P}(\mathbf{x}) d^n x, \quad (18)$$

$\rho_{i,j}$ being the linear correlation coefficients (which fulfill the condition $|\rho_{i,j}| \leq 1$). In particular, if the quantities x_1, x_2, \dots, x_n are independent, the covariance matrix is diagonal, its non-zero elements being the squares of the considered quantities standard errors (their dispersions).

If we make a set of N measurements under identical circumstances, and we have as a result the variables $\mathbf{x}(1), \mathbf{x}(2), \dots, \mathbf{x}(N)$, the estimates of the real values and of the standard errors for individual or mean values are given by the relations (14 – 16). If, on basis of the performed measurements, we evaluate a parameter expressed by a function $F(\mathbf{x})$, then, in order to estimate its true value and standard error, we must first evaluate the relative covariances $\left| \frac{\Gamma_{i,j}}{a_i a_j} \right|$. If all these relative covariances

are much smaller than the unit, then the true value of the quantity F is estimated by

$$\text{Est}_F \equiv \tilde{F} = F(\tilde{\mathbf{x}}) \quad (19)$$

and the standard error is evaluated by the error propagation Gauss formula

$$\sigma_F^2 = \sum_{i,j=1}^n \left(\left. \frac{\partial F}{\partial x_i} \right|_{\tilde{\mathbf{x}}} \sigma_i \right) \left(\left. \frac{\partial F}{\partial x_j} \right|_{\tilde{\mathbf{x}}} \sigma_j \right) \rho_{i,j}, \quad (20)$$

where the linear correlation coefficients are determined by the relation

$$\rho_{i,j} = \frac{\sum_{k=1}^N (x_i(k) - \tilde{x}_i)(x_j(k) - \tilde{x}_j)}{\sqrt{\left[\sum_{k_i=1}^N (x_i(k_i) - \tilde{x}_i)^2 \right] \left[\sum_{k_j=1}^N (x_j(k_j) - \tilde{x}_j)^2 \right]}} \quad (21)$$

and the standard errors by the relations (15) and (16), respectively. If at least one relative covariance is not small enough, then we define

$$F(i) \equiv F(\mathbf{x}(i)) \quad (22)$$

and we use the relations (14-16).

Generally, a quantity is affected by both systematic and random errors. In this case, the total error will be evaluated through the error propagation formula, as

$$s_x = \sqrt{\sigma_x^2 + \delta_x^2}. \quad (23)$$

Obviously, this relation allows us to establish in what case we can use but one type of error: when the other type is much smaller. Therefore, if we make several measurements and the differences between them are much bigger (smaller) than the reading (systematic) errors, this means that we can use only the random (systematic) errors. What is to be remembered is that the relations (20) and (23) will also be used in case some of the x_i quantities are discrete, case when the standard errors of the respective quantities are evaluated with the relations (9) or (10). For instance, let us consider the case of the counting speed of a radiation detector with the dead time τ . If we measure N_S impulses in the presence of the radioactive

source during the time interval t_S , and N_B impulses respectively for the laboratory radiation background during the time interval t_B , then the counting speed for the source will be

$$n = \frac{N_S}{t_S - N_S\tau} - \frac{N_B}{t_B - N_B\tau} \quad (24)$$

and its standard error will be

$$\sigma_n = \sqrt{\frac{N_S}{t_S(t_S - N_S\tau)} + \frac{N_B}{t_B(t_B - N_B\tau)}}, \quad (25)$$

because the relative errors for the time measurements, as well as the corresponding relative covariances, are negligible, so that we can use the Poisson distribution. In exchange, if we repeat the measurement under identical circumstances, its mean value and its standard error will be calculated with the relations (14 – 16), because the counting speed is a continuous quantity.

Finally, let us analyze the case of the evaluation of a parameter from the relation between two physical quantities. Most of the relations encountered (practically, all those encountered in the didactic laboratories) are linear or can be brought to this form. Thus, a relation of the form $y = a + b \cdot f(x)$, where a and b are the parameters to be determined, and $f(x)$ is a known function (completely determined by the measured value of x) can be brought to the linear form with the help of the substitution $X = f(x)$. A relation of the form $y = a \exp(bx)$ can be turned linear with the help of the substitution $Y = \log y$ (the graph $Y = Y(x)$ is a (simple) logarithmic scale representation, see Chapter II). A relation of the form $y = a \cdot x^b$ can be also linearized with the help of the substitutions $Y = \log y$ and $X = \log x$ (the graph $Y = Y(X)$ is a double logarithmic scale

representation, see Chapter II). Consequently, we will analyze the method of determining the parameters m and n from the relation

$$y = mx + n, \quad (26)$$

where $m = \frac{\Delta y}{\Delta x}$ is the slope and $n = y|_{x=0}$ the ordinate at the origin (the abscissa at the origin being, of course, $x|_{y=0} = -\frac{n}{m}$).

Let us consider the set of experimental pairs of data $\{x(i), y(i) | i = \overline{1, N}\}$ and let us define the expression

$$F(m, n) = \frac{1}{N-1} \sum_{i=1}^N (y(i) - mx(i) - n)^2. \quad (27)$$

It can be observed that this expression is equal to the square of the standard error for an experimental value of the quantity y with respect to the straight line (26). Under these circumstances, the best choice for the parameters m and n is the one that minimizes the function $F(m, n)$. By deriving the function with respect to m and n and by canceling the derivatives, we obtain

$$m = \frac{\sigma_y}{\sigma_x} \rho_{x,y}, \quad n = \tilde{y} - m\tilde{x}, \quad (28)$$

where the mean values, standard errors and linear correlation coefficient are calculated with the relations (14), (15) and (21). What is to be remembered is that the value of the linear correlation coefficient is a hint about the accuracy of the use of Eq. (26). Indeed, if the modulus of the coefficient is smaller than 0.5, then the quantities x and y are practically non-correlated, and if the modulus of the coefficient is situated between 0.5 and 0.9, then the quantities x and y are correlated, but not linearly. A

good linear correlation is characterized by a modulus of the correlation coefficient greater than 0.95.

If we replace the values of the parameters m and n calculated with the expressions (28) in the relation (27), we will obtain the standard error of any value of the quantity y given by the equation (26), in particular of the ordinate at the origin n

$$\sigma_n = \sigma_y \sqrt{1 - \rho_{x,y}^2} = \sqrt{\sigma_y^2 - m^2 \sigma_x^2} \quad (29)$$

(obviously, the standard error of the abscissa at the origin will be $\sigma_x \sqrt{1 - \rho_{x,y}^2}$). In order to calculate the standard error of the slope m , let us observe that, if we divide the equation (26) by x (while eliminating from the set of the experimental values of the pair corresponding to the zero value for x , if this value was measured), we will also obtain a linear relation, this time between $\frac{1}{x}$ and $\frac{y}{x}$:

$$\frac{y}{x} = n \frac{1}{x} + m, \quad (30)$$

where the roles of the parameters m and n are reversed; therefore

$$\sigma_m = \sigma_{y/x} \sqrt{1 - \rho_{1/x, y/x}^2} = \sqrt{\sigma_{y/x}^2 - n^2 \sigma_{1/x}^2}. \quad (31)$$

In the case when the analyzed relation cannot be reduced to a linear shape, the unknown parameters will be determined with the help of computers, by using one of the numerous fitting programs that may be found.

A more special case is that of an extremum determination. If we have a relation $y = f(x)$ with a relatively slight variation and, in a restricted domain of values for x , a clear-cut extremum for y , this can be very well described by the Lorentz probability distribution

$$P(x) = \frac{1}{\pi\sigma_x} \cdot \frac{4\sigma_x^2 x^2}{(x^2 - a_x^2)^2 + 4\sigma_x^2 x^2}, \quad (32)$$

where $x \geq 0$ and $a_x \gg \sigma_x > 0$ (when $x < 0$, $a_x < 0$, a similar analysis can be made; in the case when $a_x = 0$, Eq. (32) becomes

$$P(x) = \frac{1}{\pi\sigma_x} \cdot \frac{\sigma_x^2}{x^2 + \sigma_x^2}, \text{ where } -\infty < x < \infty). \text{ It can be seen that, like in the}$$

Gauss's distribution case, a_x is the most probable value for the variable x

$$(P(a_x) = P_{\max} = \frac{1}{\pi\sigma_x}), \text{ the limits of the domain of values for } x \text{ being the}$$

least probable ones (actually, they are impossible, $P(0) = P(\infty) = P_{\min} = 0$).

More than that, it can also be seen that the equation

$$P(x) = \frac{P_{\max} + P_{\min}}{2} = \frac{1}{2\pi\sigma} \text{ has the solutions } x_{1,2} = \sqrt{a_x^2 + \sigma_x^2} \pm \sigma_x,$$

satisfying the condition $\frac{|x_1 - x_2|}{2} = \sigma_x$. Therefore, in this situation, the

standard error for the position of the function $y = f(x)$ extremum is given

by the half of the difference between the positions of the points for which the equality

$$f(x) = \frac{y_{\max} + y_{\min}}{2} \quad (33)$$

is satisfied. Here $y_{\max} - y_{\min}$ is the variation of the function in the region of the extremum taken into consideration.

3. The rough errors have as their causes either the observer's lack of attention or some accidental malfunction of the measuring device and must be eliminated from calculations. Generally, this is easy to do, because

these values are strongly different from the others. However, it is good to define accurate criteria for eliminating the rough errors.

Let us consider the case of a continuous parameter x . According to the normal distribution, the probability of obtaining, in the measuring process, a value that should not differ from the true value a_x by more than

$\zeta_x \sigma_x$ ($\zeta_x = \frac{x - a_x}{\sigma_x}$ being the reduced deviation of the quantity x) is

given by the probability integral

$$\Phi(\zeta_x) = \sqrt{\frac{2}{\pi}} \int_0^{\zeta_x} \exp\left(-\frac{z^2}{2}\right) dz \quad (34)$$

and it is called a confidence level. Just as an additional information, $\Phi(1) = 0,6827$, $\Phi(2) = 0,9545$ and $\Phi(3) = 0,9973$.

The choice of the confidence interval for an individual value $x(i)$, defined as $[\tilde{x} - \zeta_{x(i)} s_{x(i)}, \tilde{x} + \zeta_{x(i)} s_{x(i)}]$, where $s_{x(i)}$ is the total error that affects the individual value $x(i)$, given by the relation (23), is made on the basis of the condition

$$\Phi\left(\zeta_{x(i)}\right) + \frac{\zeta_{x(i)} s_{x(i)}}{x(i)} = 1 \quad (35)$$

(if $x(i) = 0$, then the half-width of the corresponding relative confidence interval, $\frac{\zeta_{x(i)} s_{x(i)}}{x(i)}$, will be replaced by the average of the half-widths of the intervals for the neighboring individual values). Then, if an individual value $x(i)$ does not fit in the confidence interval, this means that $x(i)$ is a rough error and it must be eliminated from the calculations.

Obviously, equation (35) is a transcendental one, and it can only be solved numerically. When there is no such possibility, we can choose a conventional value for the level of confidence and therefore for all the

confidence intervals. The usual choice for the reduced deviation is the value $\zeta_{x(i)} = 3$; the criterion for rough error elimination thus obtained in this way is known as the 3σ (Massey) criterion.

When the rough errors are eliminated, the mean value and the standard error are recalculated and the rough error elimination criterion is reapplied. The process will be repeated until all the remaining values satisfy the criterion.

In the case of linear correlations, the conditions for the integral of the probability density (17) give the chosen confidence level is

$$\begin{aligned} \left(\frac{x - x(i)}{\zeta_{x(i)} s_{x(i)}} \right)^2 + \left(\frac{y - y(i)}{\zeta_{y(i)} s_{y(i)}} \right)^2 - 2\rho_{x,y} \left(\frac{x - x(i)}{\zeta_{x(i)} s_{x(i)}} \right) \left(\frac{y - y(i)}{\zeta_{y(i)} s_{y(i)}} \right) \\ = 1 - \rho_{x,y}^2, \end{aligned} \quad (36)$$

where the reduced deviations for x and y are evaluated with the relation (35) (or defined by the 3σ criterion), and the total errors of the individual values depend on the fact that an experimental point $(x(i), y(i))$ can be measured several times, under identical circumstances. The equation (36) defines a confidence ellipse. If the point with the coordinates $(x(i), y(i))$ belongs to the straight line (26), this one must intersect with the confidence ellipse. Obviously, the intersection condition will be reduced to a second order equation, which admits real solutions if and only if its discriminant is positive, that is if

$$\begin{aligned} \left(\frac{y(i) - mx(i) - n}{\zeta_{y(i)} s_{y(i)}} \right)^2 \\ \leq 1 - \rho_{x,y}^2 \frac{\zeta_{x(i)} s_{x(i)}}{\zeta_{y(i)} s_{y(i)}} \cdot \frac{\sigma_y}{\sigma_x} \left(2 - \frac{\zeta_{x(i)} s_{x(i)}}{\zeta_{y(i)} s_{y(i)}} \cdot \frac{\sigma_y}{\sigma_x} \right), \end{aligned} \quad (37)$$

where σ_x , σ_y are the errors for the entire set of experimental points, given by the relation (15). In particular, if

$$\frac{\zeta_{x(i)}s_{x(i)}}{\sigma_x} \approx \frac{\zeta_{y(i)}s_{y(i)}}{\sigma_y} \equiv \zeta(i), \quad (38)$$

the condition (37) becomes

$$|y(i) - mx(i) - n| \leq \zeta(i)\sigma_n. \quad (39)$$

If, we also fix the value of the reduced deviation $\zeta_{y(i)}$ by means of the condition

$$\zeta_{y(i)}^{-2} + \rho_{x,y}^2 = 1 \quad (40)$$

(for instance, the condition $\zeta_{y(i)} = 3$ is equivalent to a linear correlation coefficient $\rho_{x,y} = 0,9428$, while $\rho_{x,y} = 0,95$ implies $\zeta_{y(i)} = 3,2026$), then the condition (37) becomes

$$|y(i) - mx(i) - n| \leq s_{y(i)}, \quad (41)$$

condition that can immediately be generalized for an arbitrary dependence $y = f(x)$ under the form

$$|y(i) - f(x(i))| \leq s_{y(i)}. \quad (42)$$

II. PRESENTATION OF THE EXPERIMENTAL RESULTS

The presentation of experimental results in a report will be made in agreement with a set of rules:

1. **All** measured data **must** be present in the paper.
2. **All** measured data **must** be expressed in the **International System** units, their multiples or sub-multiples, or in tolerated units, under the form $x = \{x\}\langle x \rangle$, where x is the physical quantity, $\{x\}$ is its

numerical value, and $\langle x \rangle$ is its measure unit. If it is necessary to use an exponential format for the numerical value, only one non-null figure will be written before the decimal point. For instance, the value $U = 0.00006563 \text{ V}$ will be written either as $U = 6,563 \cdot 10^{-5} \text{ V}$ or $U = 65,63 \mu\text{V}$.

3. **All** sets of experimental data, as well as those calculated for each experimental point alone, will be presented in **Tables**. The Table head must contain for each line (column) the name of the physical quantity and in parenthesis, the used measure unit, in the form: $x(\langle x \rangle)$. In the case when the exponential format is used, the order of magnitude will also be introduced. For the previous example, the form will be either $U(10^{-5} \text{ V})$ or $10^5 U(\text{V})$ respectively, the corresponding numerical value from the table being 6.563, or $U(\mu\text{V})$, the corresponding numerical value being 65.63.
4. In the case when the scale of the measuring device is not directly graded in IS units or in their multiples or sub-multiples, in the Table will appear two lines (columns), the first with the measured values expressed in **divisions**, and the second with the values expressed in **IS units**. This supplementary line (column) can be left out only when the dimension of the respective quantity does not directly interfere in the calculations of the final results.
5. For **all** the used devices, **the scale factor** will also be mentioned in the report, under the form $1 \text{ div} |_{\text{x}} = \{x_{\text{div}}\} \langle x \rangle$. These factors are necessary not only for transforming the divisions in IS values, but also for evaluating the systematic errors.

6. ***The calculation of errors will be made for all the obtained results.***

The final results will be expressed in the form $x = (\{\tilde{x}\} \pm \{s_{\tilde{x}}\})(x)$.

The number of decimals is determined by the condition that the last two should be affected by the error. For instance, if the value obtained in IS units is 745.336286735, and the value of the error, in the same units, is 0.00891467668, the result will be presented in the rounded form 745.3363 ± 0.0089 .

7. ***For all the studied correlations there will be graphs made on millimetric paper.*** These graphs have to obey the following rules:

- i. The size of a graph must be **at least** A5 (half of an A4 page format), and the length / width ratio must fit between $2/3$ and $3/2$.
- ii. At the ends of the coordinate axes will be written the physical quantities and the units of measure, the same as in the heads of each Table.
- iii. The axes must not necessarily intersect at the origin. If, for instance, the experimental values lie between 23.89 and 24.44, the corresponding axis must include values between 23.85 and 24.45.
- iv. **The experimental values will not be marked on the axes.** They do appear in Tables. On the axes will only be marked round values, thus allowing the easy reading of any point from the graph. In the previous example, on the axes will be written values in paces of 0.05 or 0.1 (that is 23.85; 23.90; 23.95 etc. or 23.85; 23.95; 24.05 etc.).
- v. If necessary, either for the linearization of a correlation, or because the represented quantity varies by several orders of

magnitude, representations in simple logarithmic scale (only one logarithmic quantity) or double logarithmic scale (both logarithmic quantities) will be used. This means that on the axis will be written the quantity x (with its unit and rounded values), but the distances between these rounded values will be measured proportionally with the logarithm of their ratio (therefore, on the axis, $\log x$ – natural or decimal, following the necessities – will be marked).

- vi. On the graph there appear **all** the experimental points (rough errors included), with error bars (vertical bars, varying from $y(i) - s_{y(i)}$ to $y(i) + s_{y(i)}$, and, if necessary, similar horizontal bars too). The curve **must not** go through the points, but through the confidence ellipses (or, in the first approximation, through the error bars), except for the points (error bars) that correspond to rough errors. The only graphs that must go through **all** the points (bars of errors) **without** tests for eliminating the rough errors, are the calibration (gauge) curves.
- vii. **In the case of linear representations, the slope of the straight line m will not be mistaken for the tangent of the angle formed by it with the abscissa, $\tan \alpha$.** The slope of the straight line is a physical quantity, with a measure unit, and that depends only upon the experimental results, while the tangent of the angle formed by the straight line with the abscissa is a non-dimensional number and depends upon the representation scale chosen for the graph.
- viii. If the linear relation is only a first approximation, especially valid for certain values of the parameter on the abscissa (for

instance, for small values of the parameter) the experimental curve will be presented, and the parameters of the straight line searched for are given by those of the tangent at the curve in the maximum precision domain (in the suggested example, the tangent at the origin). In order to evaluate the errors, there will be made several sets of measurements, and then will be calculated the mean and standard error of the slope and/or of the ordinate (abscissa) at the origin.

- ix. The results evaluated from the graph (slopes, ordinates and/or abscissas of certain points, respectively) are **not** present on the graph, but in the text of the paper, together with the other results.
- x. The graph of a discrete quantity is **not** a continuous curve, but a histogram (a graph in steps).
- xi. Graphs are drawn in pencil, in order to be easily corrected.
- xii. If on a graph there are several curves, they are to be drawn in different colors (the experimental points included), in order to be easily differentiated, and in a corner of the graph there will be inserted a **caption** (a short segment from each color, with the explanation of the curve (the values of the parameters) drawn in that color). If one has no colored pencils, the experimental points belonging to different curves will be marked with different signs, while the caption will contain those signs instead of the colored segments.