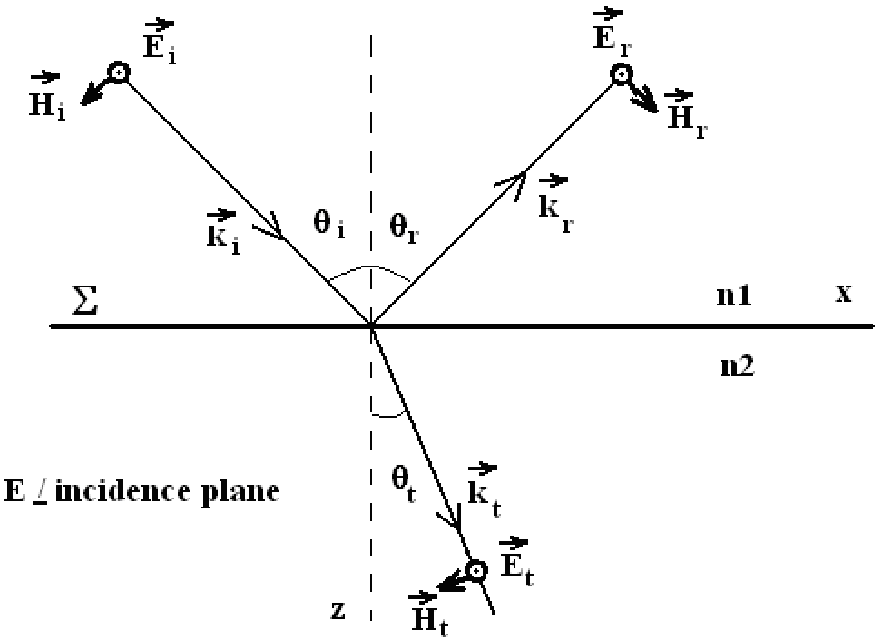
Fresnel relations

(electric field normal to the incident plane the *s-case*)

Geometry for the electric field normal to the incident plane:



Electric field ⊥ to the incident plane

If $\vec{\mathbf{E}}_i$ // incident plane, the magnetic field is normal to this plane. The figure can be deduced from the one above, keeping in mind that $\vec{\mathbf{E}} \times \vec{\mathbf{H}} // \vec{\mathbf{k}}$.

$$\vec{\mathbf{E}}_i = E_i \vec{\mathbf{u}}_y \qquad \vec{\mathbf{H}}_i = \frac{1}{Z_i} E_i \vec{\mathbf{u}}_i \times \vec{\mathbf{u}}_y$$

where:

$$\vec{\mathbf{u}}_i = \sin\theta_i \, \vec{\mathbf{u}}_x + \cos\theta_i \, \vec{\mathbf{u}}_z$$

The continuity of the tangential components of the electric field writes as:

$$E_{0i} + E_{0r} = E_{0t}$$

and that of the magnetic field is:

$$(H_{0r} - H_{0i})\cos\theta_i = -H_{0t}\cos\theta_t$$

or:

$$\left(E_{0i} - E_{0r}\right)\frac{\cos\theta_i}{Z_1} = E_{0t}\,\frac{\cos\theta_t}{Z_2}$$

Ratios:

$$r_{perp} = \left(\frac{E_{0r}}{E_{0i}}\right)_{perp} \qquad t_{perp} = \left(\frac{E_{0t}}{E_{0i}}\right)_{perp}$$

are known as the Fresnel coefficients for reflection and transmission in the perpendicular case. They are obtained immediately from the above relations:

$$r_{perp} = \frac{Z_2\cos\theta_i - Z_1\cos\theta_t}{Z_2\cos\theta_i + Z_1\cos\theta_t} = \frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t} = -\frac{\sin\left(\theta_i - \theta_t\right)}{\sin\left(\theta_i + \theta_t\right)}$$

$$t_{perp} = \frac{2Z_2\cos\theta_i}{Z_2\cos\theta_i + Z_1\cos\theta_t} = \frac{2n_1\cos\theta_i}{n_1\cos\theta_i + n_2\cos\theta_t} = \frac{2\cos\theta_i\sin\theta_t}{\sin\left(\theta_i + \theta_t\right)}$$

The equalities are exact in non-magnetic materials, where $\frac{Z_1}{Z_2} = \frac{n_2}{n_1}$.

For the parallel situation, $\vec{\mathbf{E}}_i$ // incident plane, the results are:

$$r_{//} = \left(\frac{E_{0r}}{E_{0i}}\right)_{//} \qquad t_{//} = \left(\frac{E_{0t}}{E_{0i}}\right)_{//}$$

$$r_{//} = \frac{Z_1 \cos\theta_i - Z_2 \cos\theta_t}{Z_1 \cos\theta_i + Z_2 \cos\theta_t} = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$t_{//} = \frac{2Z_2 \cos\theta_i}{Z_1 \cos\theta_i + Z_2 \cos\theta_t} = \frac{2n_1 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t} = \frac{2\cos\theta_i \sin\theta_t}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

Exercise 1. Show that:

$$\frac{n_2}{n_1} t_{//} - r_{//} = 1 \qquad\qquad t_{perp} - r_{perp} = 1$$

Remark: The above exercise shows that some Fresnel coefficients could be >1, e.g. if r_{perp} is positive $t_{perp} = 1 + r_{perp} > 1$. This means the transmitted amplitude E_{0t} is larger than the incident one. Is this impossible ?

Energy transfer by reflection and transmission

The reflection factor is defined by:

$$R = \frac{\text{flux of reflected energy}}{\text{flux of incident energy}}$$

The transmission factor is defined by:

$$T = \frac{\text{flux of transmitted energy}}{\text{flux of incident energy}}$$

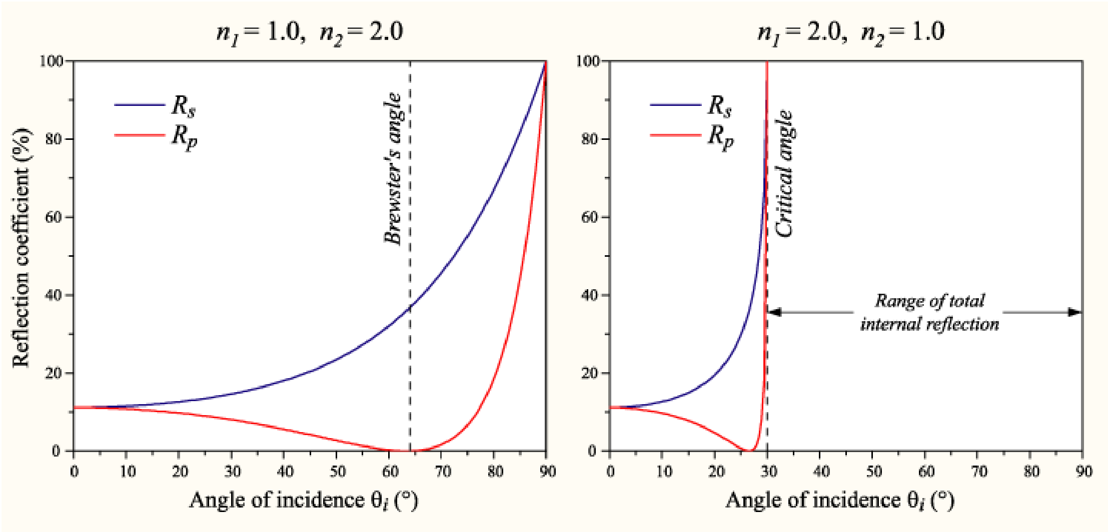
The results are:

$$R = \frac{n_1}{n_1} \frac{\vec{\mathbf{u}}_r \cdot \vec{\mathbf{u}}_n}{\vec{\mathbf{u}}_i \cdot \vec{\mathbf{u}}_n} \frac{E_{0r}^2}{E_{0i}^2} = r^2$$

$$T = \frac{n_2}{n_1}\frac{\vec{\mathbf{u}}_t \cdot \vec{\mathbf{u}}_n}{\vec{\mathbf{u}}_i \cdot \vec{\mathbf{u}}_n}\frac{E_{0t}^2}{E_{0i}^2} = \frac{n_2 \cos\theta_t}{n_1 \cos\theta_i} t^2$$

As we have neglected absorption, $T+R=1$.

From Wikipedia, the article *The Fresnel equations*



The *Brewster angle* is the angle for which there is no reflected wave with electric field parallel to the incidence plane.