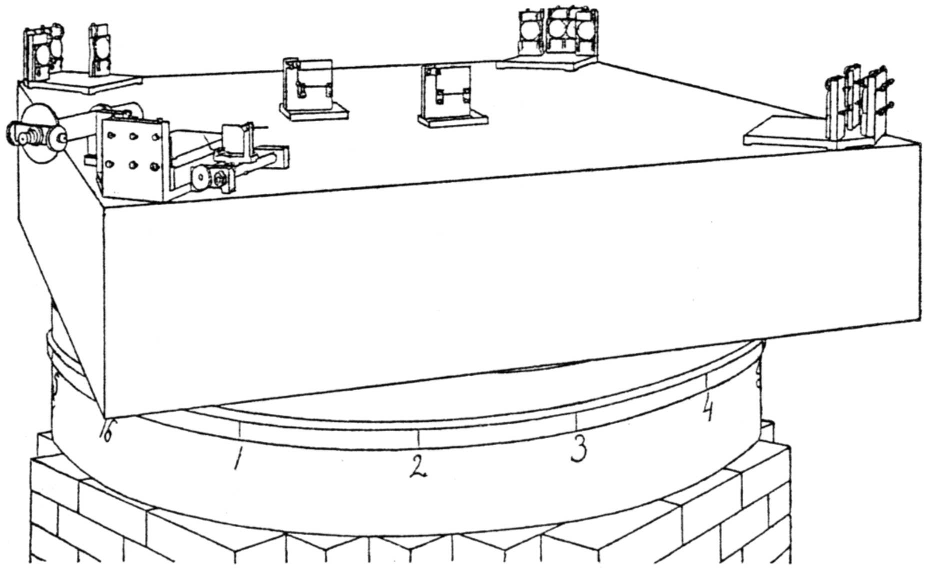
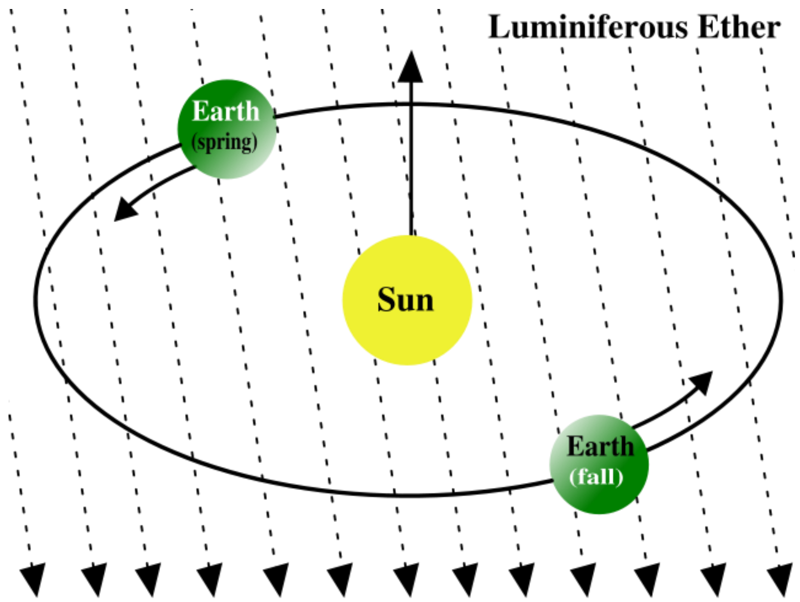
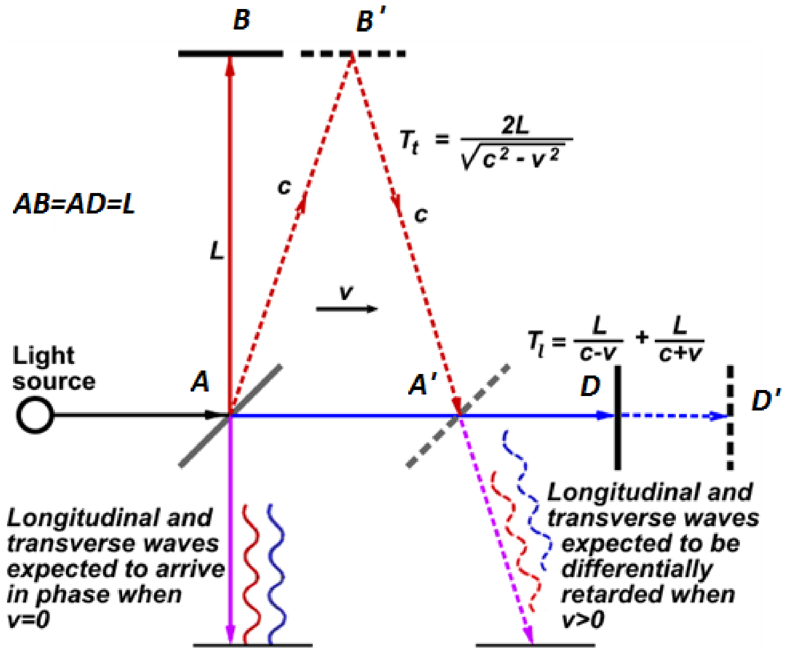
Michelson-Morley experiment
http://en.wikipedia.org/wiki/Michelson%E2%80%93Morley_experiment



Michelson and Morley's interferometric setup, mounted on a stone slab and floating in a pool of mercury



Analysis of the result



The beam traveling // $\vec{V}$: assume the beam from the source hits the beam splitter in *A* at the moment *t*=0. The transmitted beam moving with velocity *c* hits the mirror in the position *D*' after the time t_1 given by $DD' = Vt_1$. Hence $ct_1 = L + vt_1$ and $t_1 = \frac{L}{c-v}$. The backward journey is accomplished in $t_2 = \frac{L}{c+v}$. The total travel time // $\vec{V}$ is

$$T_{//} = \frac{L}{c-v} + \frac{L}{c+v} = \frac{2L}{c}\frac{1}{1-\frac{v^2}{c^2}}$$

(SR1)

For the transverse movement $\perp\vec{V}$ light propagates from *A* to *B'* during time t_3 traveling a distance ct_3. During this time the beam splitter moves the distance Vt_3=*AA*'/2. Light travels *L* in the vertical direction and Vt_3 in the horizontal direction. The inclined path from *A* to *B*' is given by: *AB'*=ct_3=$\sqrt{L^2 + V^2 t_3^2}$. Hence $t_3 = \frac{L}{\sqrt{c^2 - v^2}}$. The total travel time is

$$T_{\perp} = \frac{2L}{c}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \approx \frac{2L}{c}\left(1 + \frac{v^2}{2c^2}\right) \qquad \text{(SR2)}$$

The difference is

$$T_{//} - T_{\perp} = \frac{2}{c}\left(\frac{L}{1-\frac{v^2}{c^2}} - \frac{L}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

The corresponding path difference is $\Delta = c(T_{//} - T_{\perp})$. After rotating the experimental device with 90º, the path difference would be $-\Delta$. In terms of wavelength

$$\frac{2\Delta}{\lambda} \approx \frac{2Lv^2}{\lambda c^2} \approx 0.44 \qquad \text{(SR3)}$$

Apparatus was capable to detect shifts of less than 0.03. It was not the case.

Lorentz transformations

Principles of Special Relativity (SR)

(not requested at the exam)

1. *The velocity of light in vacuum is a universal constant which do not depend on the inertial reference system (IRS) from which it is measured.*
2. *The laws of physics are the same in all IRS.*

General properties of the Lorentz transformations

Assume two IRS (S) and (S’). (S’) moves with **V** with respect to (S) (usually along the common axes *xx*’). A point has *four* coordinates in each IRS: (*x*, *y*, *z*, *t*) or (*x’*, *y’*, *z’*, *t’*). Points are also called *events*.

For the convenience put

$$x_1 = x \quad x_2 = y \quad x_3 = z \quad x_4 = ict \qquad \text{(SR1)}$$

The wave equation in vacuum is:

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}=0 \qquad \text{(SR2)}$$

In the new notations:

$$\sum_{\mu=1}^{4}\frac{\partial^2\psi}{\partial x_\mu{}^2}=0 \qquad \text{(SR2')}$$

The Lorentz relations are the transformations from the coordinates of (S) to those of (S')

$$x_\mu^{'}=x_\mu^{'}(x_1,\,x_2,\,x_3,\,x_4) \qquad \mu=1,\,2,\,3,\,4$$

The wave equation (SR 2) must not vary during a coordinate transformation. Hence these transformations are linear, i.e. have the form:

$$x_\mu^{'}=\sum_{\nu=1}^{4}\alpha_{\mu\nu}x_\nu \qquad \mu=1,\,2,\,3,\,4 \qquad \text{(SR3)}$$

or

$$\begin{pmatrix} x_1^{'} \\ x_2^{'} \\ x_3^{'} \\ x_4^{'} \end{pmatrix}=\begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & \ldots\ldots\ldots\ldots & & \alpha_{24} \\ \ldots\ldots\ldots\ldots & & & \\ \alpha_{41} & & & \alpha_{44} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \qquad \text{(SR3')}$$

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Notations: $\beta=V/c$ $\qquad \gamma=(1-\beta^2)^{-1/2}$ (SR14)

The Lorentz matrix is:

$$\alpha = \begin{pmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{pmatrix} \qquad \text{(SR15)}$$

The transformation is:

$$\boxed{x'_1 = \gamma(x_1 + i\beta x_4) \qquad x'_2 = x_2 \qquad x'_3 = x_3 \qquad x'_4 = \gamma(-i\beta\gamma + x_4)} \quad \text{(SR16)}$$

Or in normal variables

$$\boxed{x' = \frac{x - Vt}{\sqrt{1 - V^2/c^2}} \qquad y' = y \qquad z' = z \qquad t' = \frac{t - Vx/c^2}{\sqrt{1 - V^2/c^2}}} \quad \text{(SR17)}$$

Time space and velocity in SR

Simultaneous events, clock synchronization, time dilation

As time changes in a Lorentz transformation (SR17) we must reconsider time measurement.

Model of a light watch. Choose an inertial frame (*S*). It is composed by rigid sticks to measure lengths and by clocks placed in each point to measure time. The medium between them is homogeneous and isotropic. A convenient theoretical watch is made by a source emitting short burst of light which travel between two parallel mirrors at rest. The light source is situated just near one of the mirrors. Successive reflections indicate ticks and tacks. They are separated by the same time intervals because c is a constant.

Synchronization of clocks in the same IRF (inertial reference frame). Assume a source situated in the origin emits light bursts. They travel as spherical waves and arrive at watches situated on a sphere at the same moment. We synchronize watches accordingly.

Definition: Physical quantities of a body in an IRF where this body is at rest are called *proper* or *rest* quantities: proper time, proper length, rest mass, rest energy.

Time dilation. This procedure does not ensure at all synchronization of watches in movement with respect to (*S*). On the contrary, (SR17) tells another story.

Let's study two events *A* and *B* situated in the same spatial point $x_A = x_B$, $y_A = y_B$, $z_A = z_B$, at two different times $t_B > t_A$. The proper time between these events is $\tau = t_B - t_A = \Delta t > 0$. In the IRF (*S*') moving with constant velocity *V* along the common axis *xx'* the time interval is

$$\Delta t' = t'_B - t'_A = \gamma(t_B - t_A) = \gamma\tau > \tau \qquad \text{(SR18)}$$

Time is dilated when measured from a moving system. Example: decay of unstable particles.

Temporal order of events. What if the two events happen in two different places? Imagine that the event *A* given by (x_A, t_A) is the cause of the event *B* given by (x_B, t_B) ; therefore $t_B > t_A$. Assume $x_B > x_A$ and represent the propagation velocity of the interaction by $u = \frac{x_B \text{-} x_A}{t_B \text{-} t_A}$. Could the temporal order of the two events be altered by a Lorentz transformation? Computation shows that it is not the case:

$$\left(t'_B - t'_A\right) = \gamma\left(t_B - \frac{V}{c^2}x_B - t_A + \frac{V}{c^2}x_A\right) = \gamma\left(t_B - t_A\right)\left(1 - \frac{uV}{c^2}\right) \qquad \text{(SR19)}$$

The order in time is preserved if $V < c$ and $u < c$. Therefore **the causality is not altered by a Lorentz transformation if reference frames can not move with velocity larger than *c* and interaction can not propagates with velocities larger than *c***. We shall assume this is the case for all transformations with physical meaning. Hence **speed of light in vacuum is the maximum velocity of the physical interactions.**

Length contraction. The proper length of a rod in the IFR (*S*) is $l_0 = x_B - x_A$. In (S') the positions of the two extremities of the stick must be measured in the same moment $t'_B = t'_A$. Hence:

$$x_B - x_A = \gamma(x'_B - x'_A + \frac{V}{c_2}t'_B - \frac{V}{c_2}t'_A) = \gamma(x'_B - x'_A)$$

$$l' = {l_0}/{\gamma} = l_0\sqrt{1-\beta^2} \qquad \text{(SR20)}$$

Application: explanation of the negative result of the Michelson experiment.

Velocity composition. In SR the composition of velocities is different from the situation in pre-relativistic physics. The new law must conserve the speed of light and it must reduce to the usual Galilean law for small velocities. To deduce the relativistic relation for velocity composition we use the normal definition of velocity:

In (S) $\quad v_x = \dfrac{dx}{dt},\ v_y = \dfrac{dy}{dt},\ v_z = \dfrac{dz}{dt}$

In (S') $\quad v'_x = \dfrac{dx'}{dt'},\ v'_y = \dfrac{dy'}{dt'},\ v'_z = \dfrac{dz'}{dt'}$

Using the Lorentz relations one finds:

$$v'_x = \frac{v_x - V}{1 - \frac{v_x V}{c^2}} \quad v'_y = \frac{v_y\sqrt{1 - V^2/c^2}}{1 - \frac{v_x V}{c^2}} \quad v'_z = \frac{v_z\sqrt{1 - V^2/c^2}}{1 - \frac{v_x V}{c^2}} \qquad \text{(SR21')}$$

The inverse relations from (S') to (S) are:

$$v'_x = \frac{v_x + V}{1 + \frac{v_x V}{c^2}} \quad v'_y = \frac{v_y\sqrt{1 - V^2/c^2}}{1 + \frac{v_x V}{c^2}} \quad v'_z = \frac{v_z\sqrt{1 - V^2/c^2}}{1 + \frac{v_x V}{c^2}} \qquad \text{(SR21'')}$$

Exercises:

1. Show that if a particle moves with the velocity c its speed measured from a moving system is also c.

2. A particle moves with velocity $v_x = 0.9c,\ v_y = v_z = 0$ with respect to (S). Compute its velocity in (S') which moves with $V \equiv V_x = 0.5c$. The same problem if $V \equiv V_x = -0.5c$.

Conclusion: The velocities of bodies are always smaller than or at most equal to the velocity of light in vacuum.

Bertozzi experiment 1964

Speed and Kinetic Energy of Relativistic Electrons*†

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Using a Van de Graaff electrostatic generator and a linear accelerator, the speeds of electrons with kinetic energies in the range 0.5–15 MeV are determined by measuring the time required for the electrons to traverse a given distance. The measurements show the existence of a limiting speed in accord with the results of special relativity. The kinetic energy, determined by calorimetry, verifies that an electric field exerts a force on a moving electron in its direction of motion that is independent of its speed.

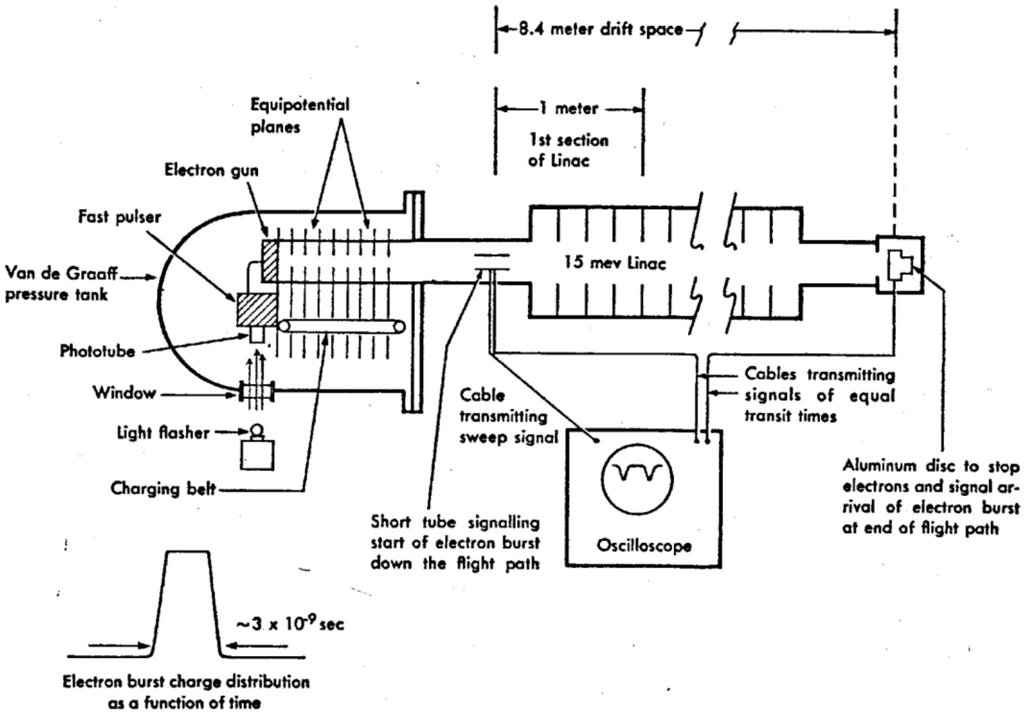

FIG. 1. Schematic diagram of the experiment set up for measuring the time of flight of the electron burst from the Van de Graaff.

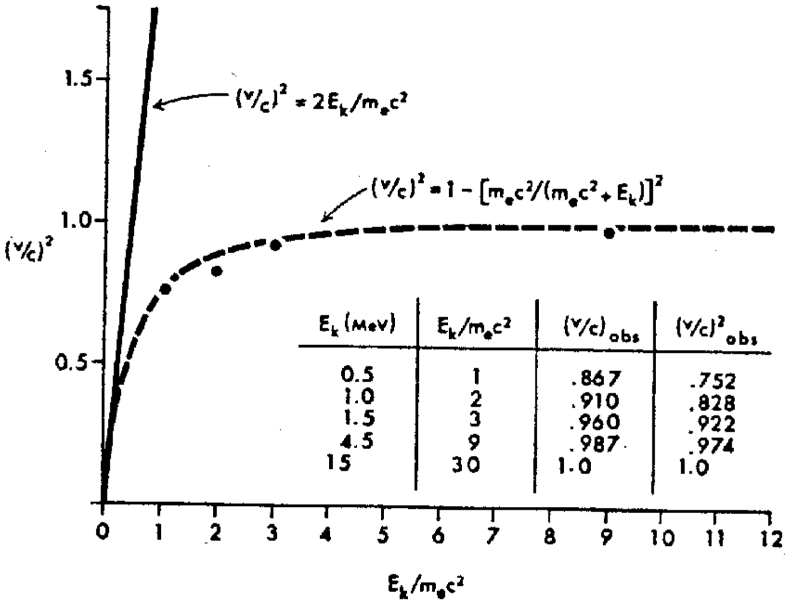

FIG. 3. The solid curve represents the prediction for $(v/c)^2$ according to Newtonian mechanics, $(v/c)^2=2E_k/m_ec^2$. The dashed curve represents the prediction of Special Relativity, $(v/c)^2=1-[m_ec^2/(m_ec^2+E_k)]^2$. m_e is the rest mass of an electron and c is the speed of light in a vacuum, 3×10^8 M/sec. The solid circles are the data of this experiment. The table presents the observed values of v/c.

From http://physics.nist.gov/cgi-bin/cuu/Value?me|search_for=atomnuc!

electron rest mass ***m_{0e}***	**electron rest energy** ***E_{0e}***
9.109 382 91 x 10^{-31} kg	**0.510 998 928 MeV**
elementary charge ***e*** **1.602 176 565 x 10^{-19} C**	