Composition of perpendicular oscillations

1. Same frequency

$x(t) = a\sin(\omega_0 t)\ \ y(t) = b\sin(\omega_0 t + \varphi_0)$ (O7)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\frac{x}{a}\frac{y}{b}\cos\varphi = \sin^2\varphi \qquad \text{(O8)}$$

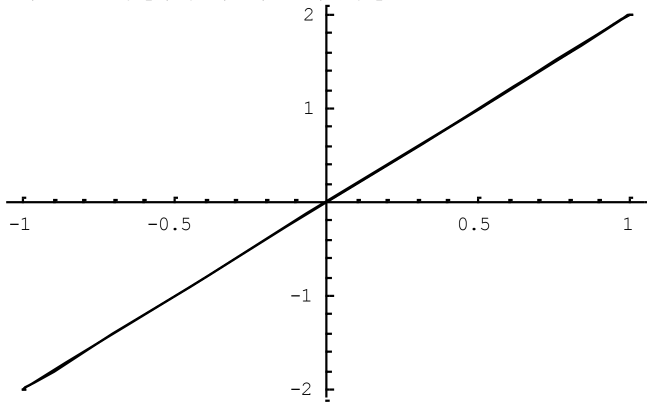
where φ is the phase difference. Particular cases:

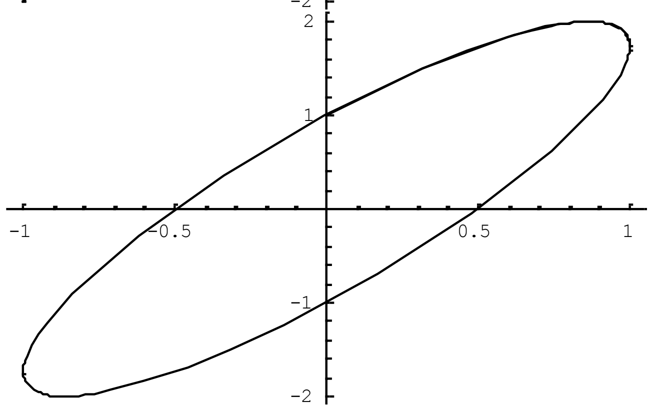
$\varphi = 0,\quad \pi \Rightarrow \frac{x}{a} = \frac{y}{b},\qquad \frac{x}{a} = -\frac{y}{b}$ $\qquad \varphi = \frac{\pi}{2} \quad \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

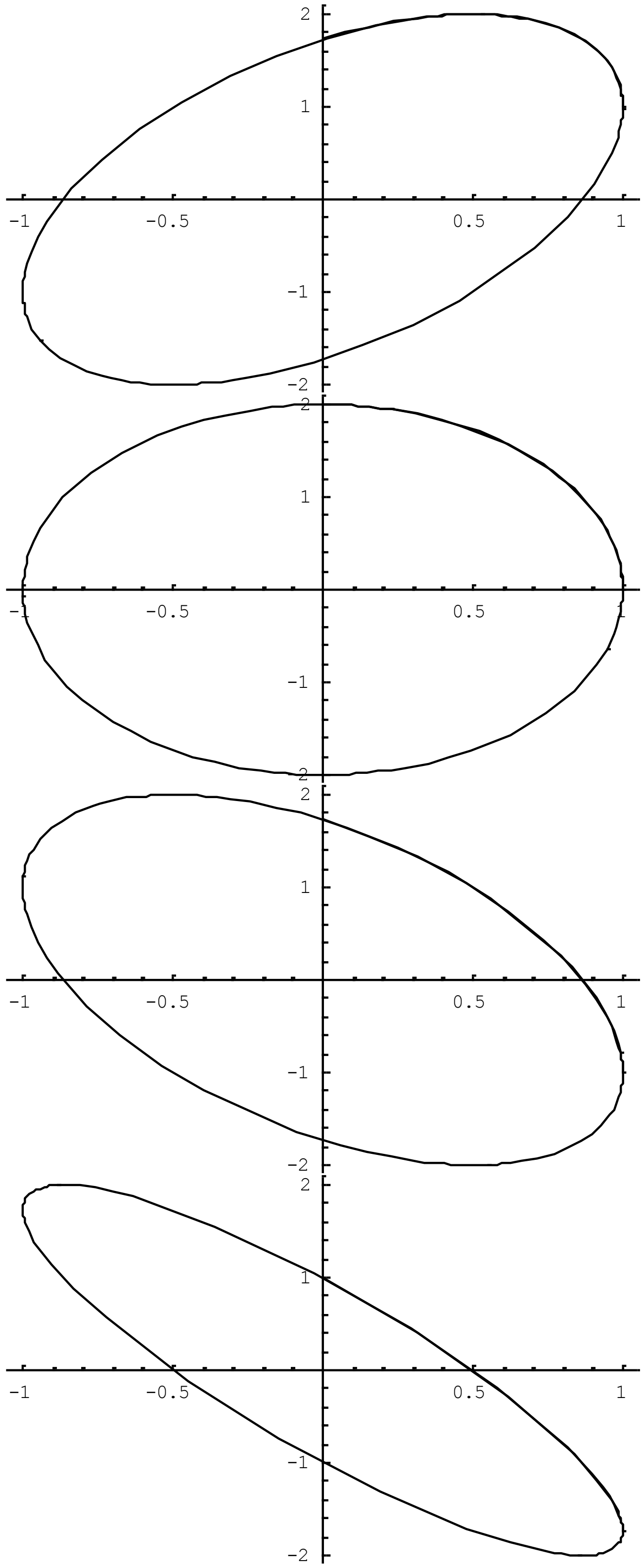
If $\varphi = \frac{\pi}{2}$ and $a = b, \quad \Rightarrow x^2 + y^2 = a^2$

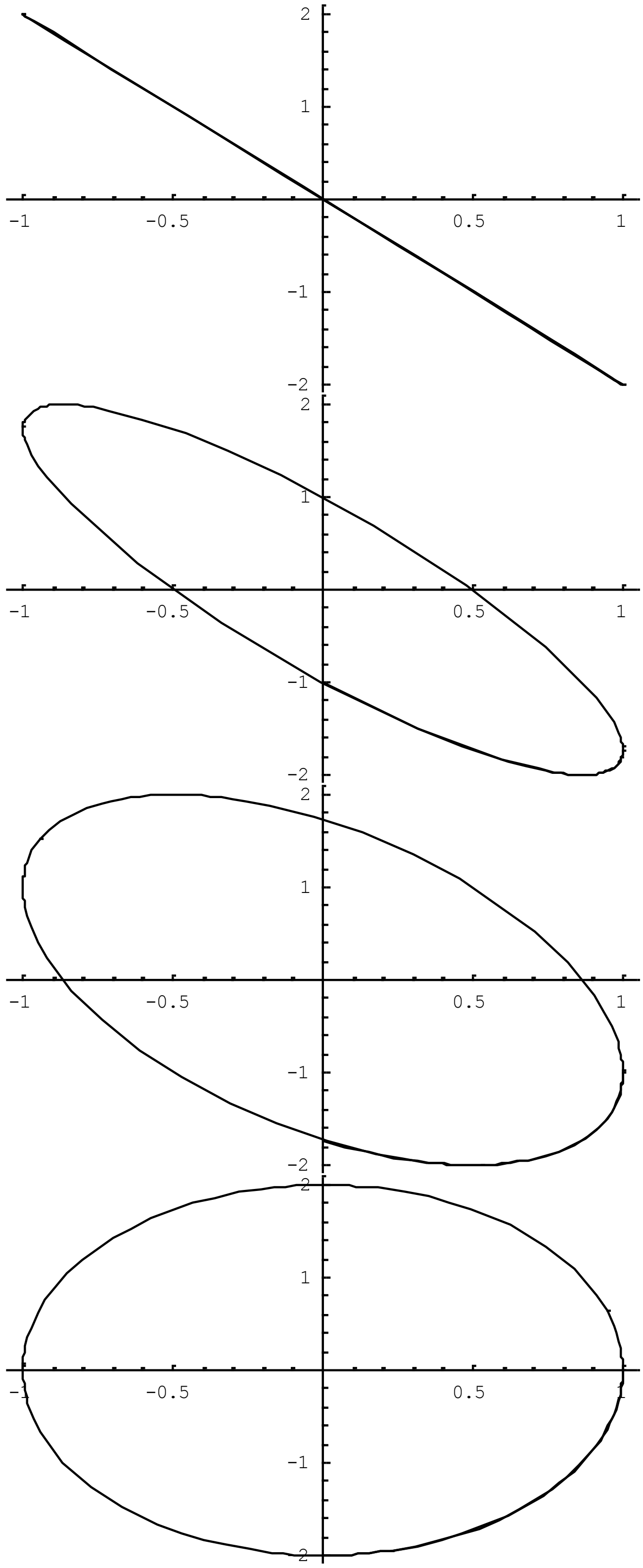
Same $\omega_0 = 5\ \text{s}^{-1}$, φ_0 from 0 to 2π, steps of $\pi/6$.

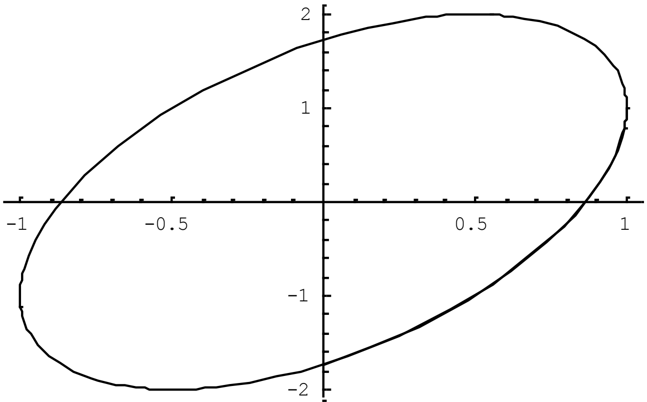
```
ClearAll[x,y,a,b,phi,om1,om2]
a=1;b=2;om1=5;om2=5;phi=Pi/6;
Do[ParametricPlot[{a*Sin[om1*t],b*Sin[om2*t+n*Pi/12]},{t,
0,Pi/2}],{n,0,24,2}]
```

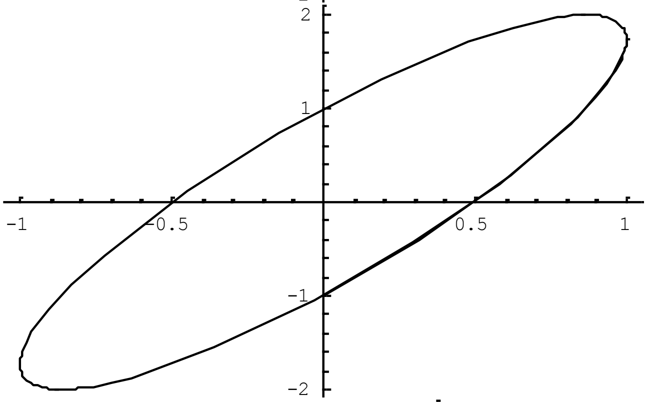


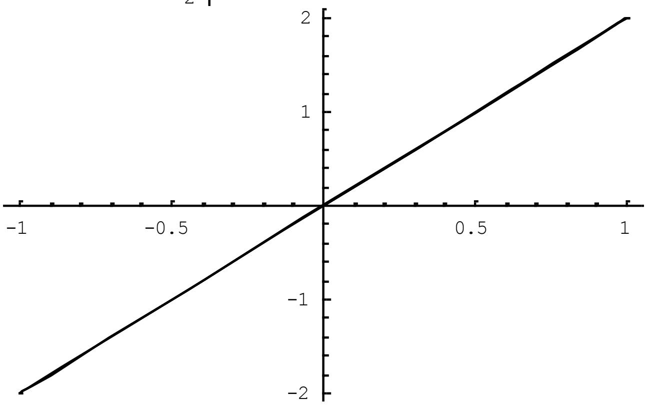



2
1
-1
-0.5
0.5
1
-1
-2


2
1
-1
-0.5
0.5
1
-1
-2







When φ grows the trajectory changes from a portion of a segment to an ellipsis and back again. The same construction describes the polarization of e.m. waves.

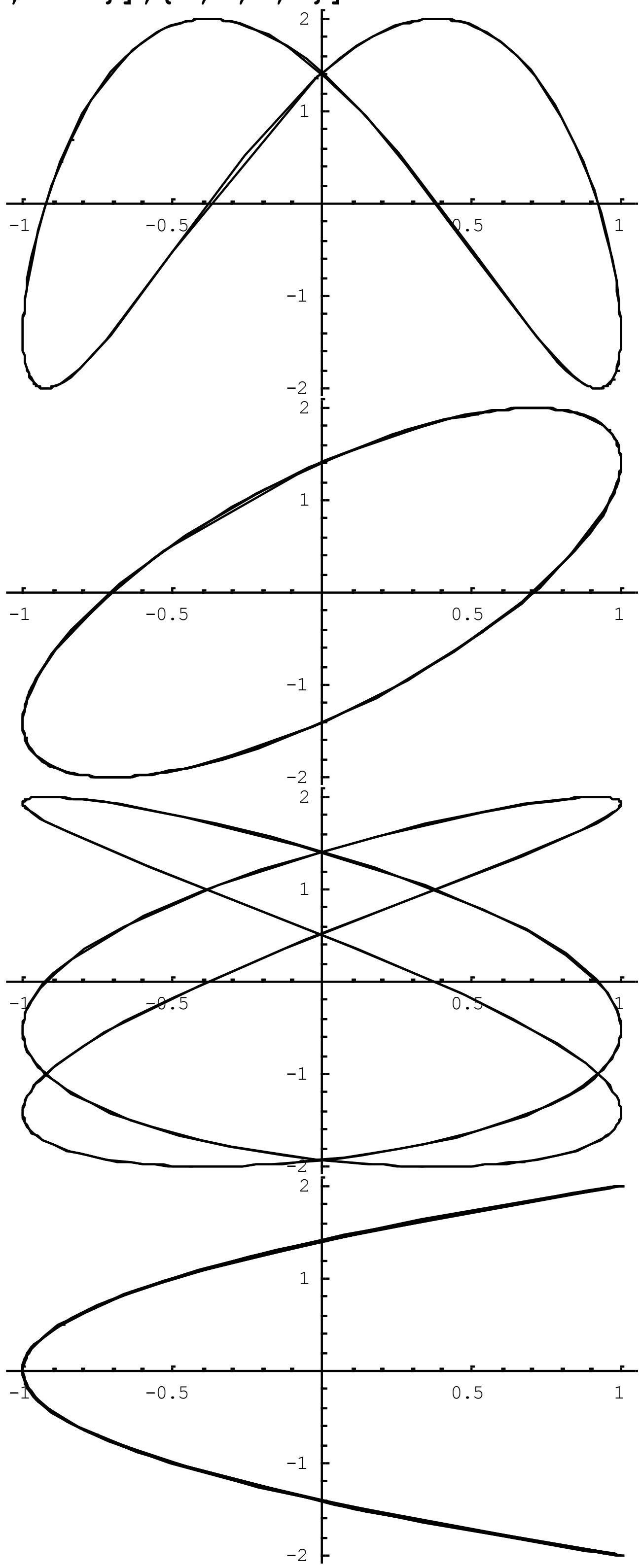
2. Different frequencies, Lissajous figures

When $\omega_1 \neq \omega_2$ the trajectory is more complicated and if ω_1/ω_2 is not rational it forms a dense set in the plane. Eq. (O7) is replaced by:

$$x(t) = a\sin(\omega_1 t) \quad y(t) = b\sin(\omega_2 t + \varphi_0) \qquad \text{(O9)}$$

ω_1 / ω_2 rational number: $\omega_{02} = 4\text{s}^{-1}$, $\omega_{01} = 2, 4, 6, 8\ \text{s}^{-1}$; $a = 1$; $b = 2$.

```
ClearAll[x,y,a,b,phi,om1,om2]
a=1;b=2;om1=2;om2=4;phi=0;
Do[ParametricPlot[{a*Sin[n*om1*t],b*Sin[om2*t+Pi/4]},{t,0
,2*Pi}],{n,1,4,1}]
```



ω_1/ω_2 irrational number; $\omega_{02} = \sqrt{2}\ \mathrm{s}^{-1}$, $\omega_{01} = 2, 4, 6, 8\ \mathrm{s}^{-1}$; $a = 1$; $b = 2$.

```
ClearAll[x,y,a,b,phi,om1,om2]
a=1;b=2;om1=2;om2=Sqrt[2];phi=Pi/4;
Do[ParametricPlot[{a*Sin[n*om1*t],b*Sin[om2*t+Pi/4]},
{t,0,20*Pi}],{n,1,4,1}]
```