Composition of parallel oscillations

$$x(t) = a_1 \sin(\omega_1 t + \varphi_{01}) + a_2 \sin(\omega_2 t + \varphi_{02}) \qquad \text{(O4)}$$

1. Same frequency

$$x(t) = a \sin(\omega t + \varphi_0)$$

with

$$a = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos(\varphi_{01} - \varphi_{02})} \qquad \text{(O5)}$$

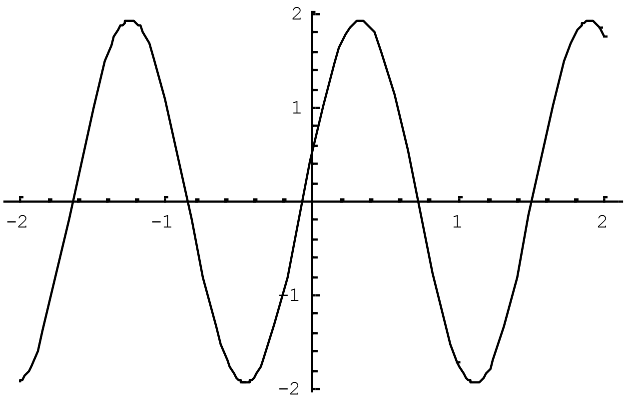
$$\tan\varphi_0 = \frac{a_1 \sin\varphi_{01} + a_2 \sin\varphi_{02}}{a_1 \cos\varphi_{01} + a_2 \cos\varphi_{02}} \qquad \text{(O6)}$$

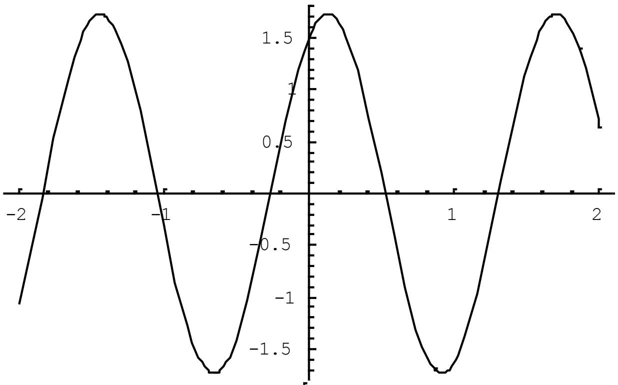
Graphic results with Mathematica

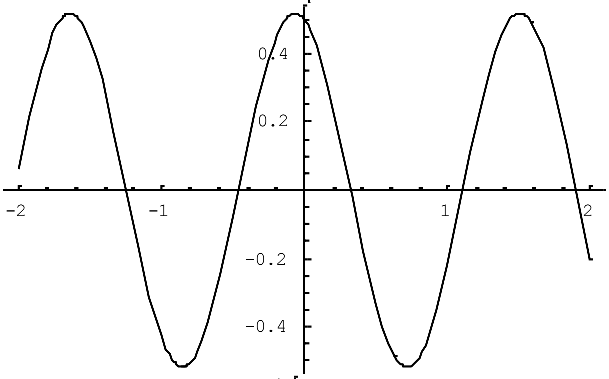
```
m=1;
x1[c1_,t_,om1_,phi1_]=c1*Sin[om1*t+phi1];
x2[c2_,t_,om2_,phi2_]=c2*Sin[om2*t+phi2];

ek1[c1_,t_,om1_,phi1_]=m*(D[x1[c1,t,om1,phi1],t])^2/2;
u1[c1_,t_,om1_,phi1_]=m*(om1*x[c1,t,om1,phi1])^2/2;
ek2[c2_,t_,om2_,phi2_]=m*(D[x2[c2,t,om2,phi2],t])^2/2;
u2[c2_,t_,om2_,phi2_]=m*(om2*x[c2,t,om2,phi2])^2/2;
```

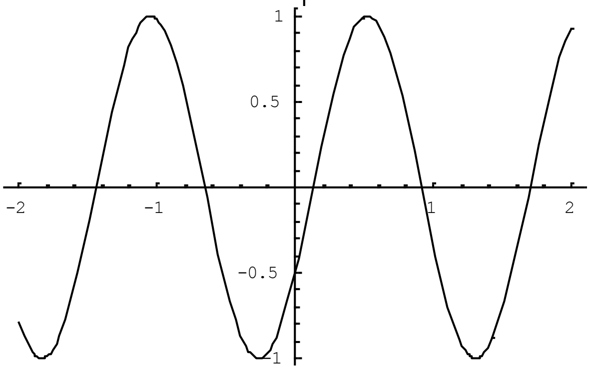
Same frequency, $\omega_0 = 4\,\mathrm{s}^{-1}$, phase shift varies between 0 and 3Pi/2, steps of Pi/2.

```
Do[Plot[x1[1,t,4,Pi/6]+x2[1,t,4,n*Pi/12],
{t,-2,2}],{n,0,18,6}]
```







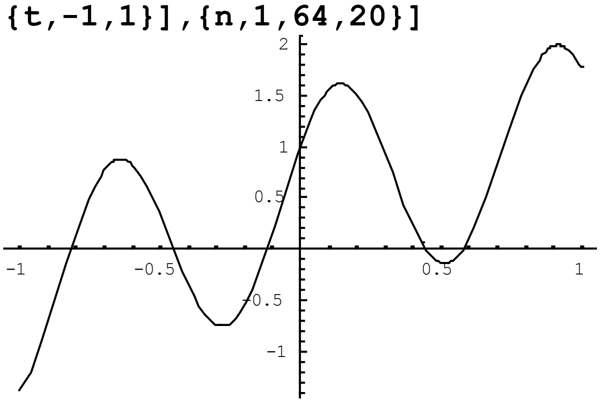


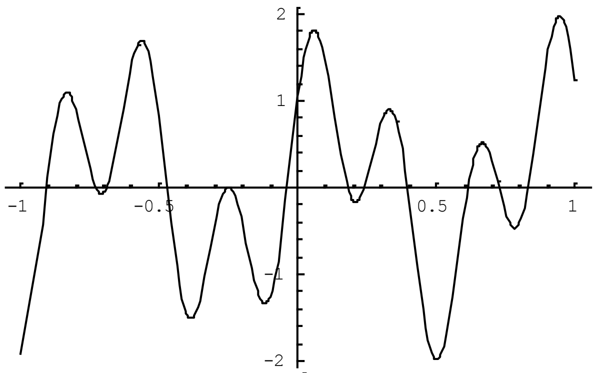
2. Different frequencies

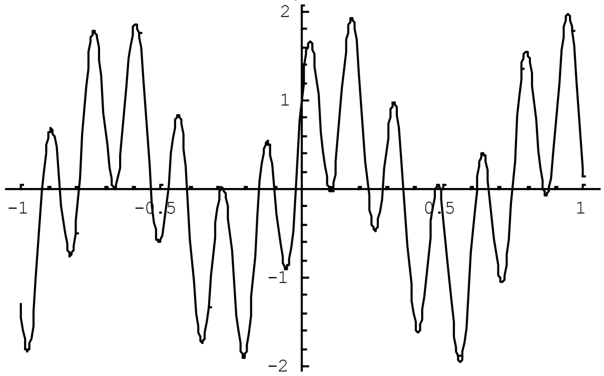
(O4) remains valid, but relations as (O5) and (O6) no more exist.

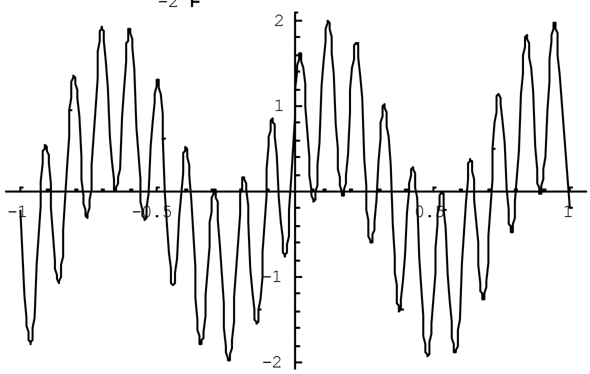
Frequency between 1 s^{-1} and 64 s^{-1} in steps of 20 s^{-1}

```
Do[Plot[x1[1,t,8,Pi/6]+x2[1,t,n,Pi/6],
{t,-1,1}],{n,1,64,20}]
```



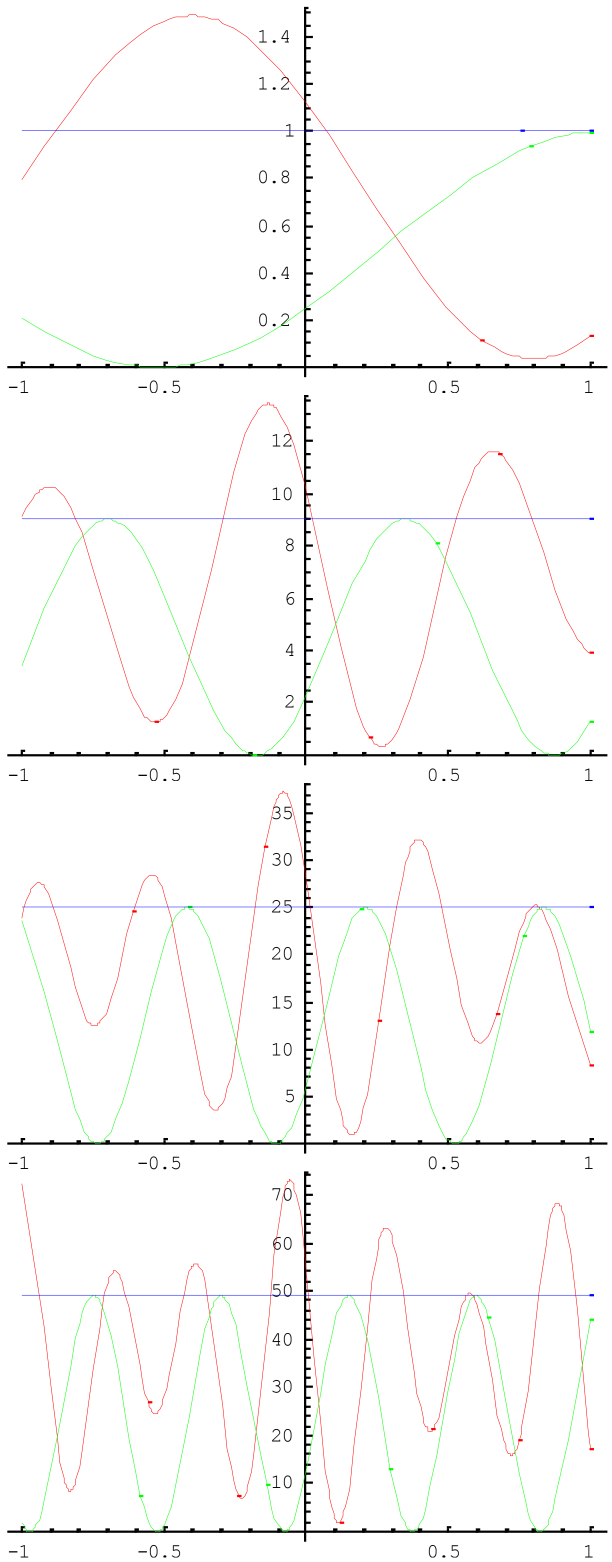




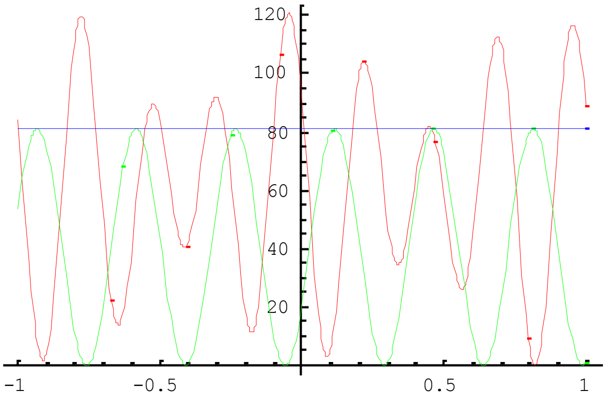


Energies: kinetic, potential, total for different frequencies: $\sqrt{2}\times n$ and n, for $n = 1, 3, 5, 7, 9$

```
Do[Plot[{ek1[1,t,Sqrt[2]*n,Pi/6]+ek2[1,t,n,Pi/6],u1[1,t,n
,Pi/6]+u2[1,t,n,Pi/6],ek1[1,t,n,Pi/6]+ek2[1,t,n,Pi/6]+u1[
1,t,n,Pi/6]+u2[1,t,n,Pi/6]},{t,-
1,1},PlotStyle→{RGBColor[1,0,0],RGBColor[0,1,0],RGBColor[
0,0,1]}],{n,1,9,2}]
```



1.4
1.2
1
0.8
0.6
0.4
0.2
-1
-0.5
0.5
1
12
10
8
6
4
2
-1
-0.5
0.5
1
35
30
25
20
15
10
5
-1
-0.5
0.5
1
70
60
50
40
30
20
10
-1
-0.5
0.5
1



120
100
80
60
40
20
-1
-0.5
0.5
1