9. Interference

1. General frame

We call *interference* the superposition of two or several waves, in particular em waves. The materials are assumed to be linear, so we can apply the superposition principle.

Interference appears under certain conditions when two or more waves superpose. We study only interference of em waves, in particular optical waves.

Question: what is light ?

When light from two sources superposes what we observe usually is an increase of the overall intensity. If the two sources are *coherent*, one observes a totally different spatial pattern formed by maxima and minima which remain stable during the observation. This is the so-called *stationary interference*. We shall deal with this type of superposition; it is the foundation of interferometry. Interferometry with elmgn waves is one of the most accurate measurement methods in physics.

Assume that in a region superpose two em waves. The total electric field is:

$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 \tag{I1}$$

The total intensity is:

$$I_{tot} = \frac{1}{2Z}\left|\vec{E}_{tot}\right|^2 \propto \left(\vec{E}_1 + \vec{E}_2\right)\left(\vec{E}^*{}_1 + \vec{E}^*{}_2\right) = I_1 + I_2 + \vec{E}_1 \cdot \vec{E}_2^* + \vec{E}_2 \cdot \vec{E}_1^* \tag{I2}$$

The interference term is $I_{12} = \vec{E}_1 \cdot \vec{E}_2^* + \vec{E}_2 \cdot \vec{E}_1^*$, hence

$$I_{tot} = I_1 + I_2 + I_{12} \tag{I3}$$

When $I_{12} \neq 0$ we have *constructive interference*, described by interference *fringes*. One says the two waves are *coherent*.

2. Coherence conditions

In real conditions one does not detect instantaneous intensities, because visible light has around 10^{14}-10^{15} Hz, so we measure an average made on 10^5-10^{13} periods. Relation (I3) has to be written:

$$\langle I_{tot} \rangle = \langle I_1 \rangle + \langle I_2 \rangle + \langle I_{12} \rangle \tag{I3'}$$

What are the conditions for a non-zero interference term ? Three conditions should be accomplished for the two waves:

- polarization along the same direction (at least to a certain extent)
- same frequency
- constant phase shift.

These are called *coherence conditions*. Such perfect coherence is hard to find in practice, but one can relax the above conditions for *partial coherent waves*.

3. Interference of two harmonic plane waves

The electric fields are parallel, so we need no more vectors:

$$E_1 = E_{01} \exp\left[i\left(\omega t - \vec{k}_1 \cdot \vec{r}_1 + \varphi_1\right)\right]$$
$$E_2 = E_{02} \exp\left[i\left(\omega t - \vec{k}_2 \cdot \vec{r}_2 + \varphi_2\right)\right]$$

The interference term is

$$I_{12} = \frac{1}{2Z} 2\sqrt{|E_{01}|^2 |E_{02}|^2} \cos\left[\left(\vec{k}_1 \cdot \vec{r}_1 - \vec{k}_2 \cdot \vec{r}_2\right) + \varphi_1 - \varphi_2\right]$$

The two waves propagate in the same material, hence $k_1 = k_2 = \dfrac{n\omega}{c}$. In optical experiments the propagation directions of both waves are almost the same, so the wave-vectors are equal as vectors:

$$\left(\vec{k}_1 \cdot \vec{r}_1 - \vec{k}_2 \cdot \vec{r}_2\right) \cong k\Delta r = \frac{n\Delta r}{\omega / c} = \frac{2\pi}{\lambda_0} n\Delta r \tag{I4}$$

where λ_0 is the wavelength in vacuum. In (I4) is apparent *the optical path difference* obtained by multiplying the path difference by the refractive index:

$$[\Delta r] = n\Delta r \tag{I5}$$

Eq. (I3') is (the time average does not change the cos function):

$$\langle I_{tot}\rangle = \langle I_1\rangle + \langle I_2\rangle + 2\sqrt{\langle I_1\rangle\langle I_2\rangle}\cos(k\Delta r + \varphi_1 - \varphi_2) \tag{I6}$$

The intensity has a sinusoidal spatial variation between maxima and minima of intensity

$$\langle I_{\max}\rangle = \langle I_1\rangle + \langle I_2\rangle + 2\sqrt{\langle I_1\rangle\langle I_2\rangle}, \qquad \text{for } k\Delta r + \varphi_1 - \varphi_2 = 2m\pi \tag{I7'}$$

$$\langle I_{\min}\rangle = \langle I_1\rangle + \langle I_2\rangle - 2\sqrt{\langle I_1\rangle\langle I_2\rangle}, \qquad \text{for } k\Delta r + \varphi_1 - \varphi_2 = (2m+1)\pi \tag{I7''}$$

The stationary interference pattern consists of a succession of surfaces definded by the condition that the phase shift should be an even number of π's for maxima and an odd number of π's for minima. Using (I4) the optical path difference is (for $\varphi_1 - \varphi_2 = 0$):

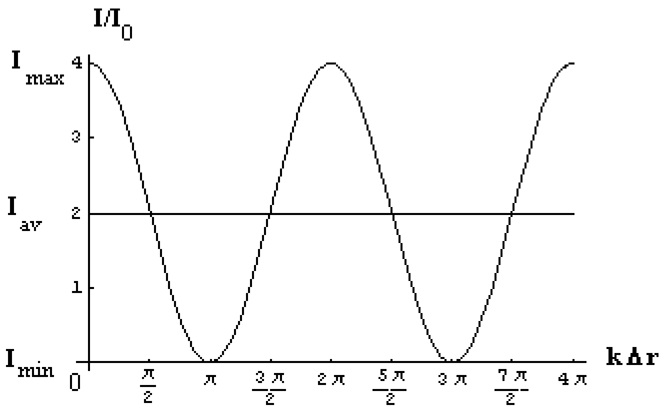
$$[\Delta r] = n\Delta r = 2m\frac{\lambda_0}{2} \qquad \text{for maxima} \tag{I8'}$$

$$[\Delta r] = n\Delta r = (2m+1)\frac{\lambda_0}{2} \qquad \text{for minima} \tag{I8''}$$

If the phase difference $\varphi_1 - \varphi_2$ has a random variation with time, maxima and minima are not observed because the interference term is averaged out.

Question: Why the phase difference $\varphi_1 - \varphi_2$ could have a random variation with time ? *Hint*: think of the atoms generating light.

Particular case: equal intensities $I_1 = I_2 = I_0$, $\varphi_1 - \varphi_2 = 0$; $I_{\max} = 4I_0$, $I_{\min} = 0$. The total intensity varies as in the following figure. The average intensity is $2I_0$.



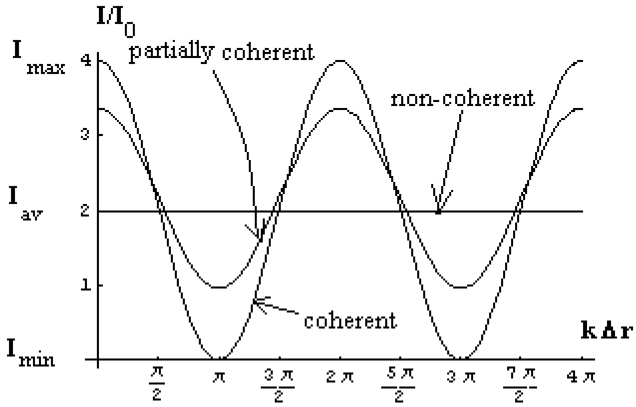
The visibility of the interference pattern is defined as

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \tag{I9}$$

For the above situation V=1

Question: Draw the intensity pattern for $I_1 = 2I_0$, $I_2 = I_0$ and compute the visibility.

Remark. Real beams waves are not harmonic (or monochromatic), but have several components of different frequencies, which form the *frequency band* of the beam. The waves are *partially coherent* and the visibility is always smaller than for the perfect coherent waves. See the figure below for the possible situations. In all three cases the average intensity is $2I_0$.



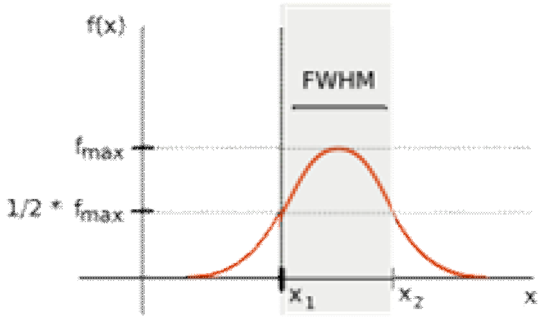
A convenient definition of the band is $\Delta\omega_{FWHM}$. The ratio between the mean frequency ω_0 and the frequency band *FWHM* is known as *the monochromaticity of the light*. For an usual source (bulb, flame, sun) $\frac{\omega_0}{\Delta\omega_{FWHM}} \approx 1-10$. For a source with a colored glass as filter $\left(\frac{\omega_0}{\Delta\omega_{FWHM}}\right)_{filter} \approx 10-20$. For a usual semiconductor laser the ratio attains easily 10^3-10^4. For a gas laser it grows to 10^6-10^9 and for a well stabilized laser it could be 10^{12}-10^{14}. That means the frequency of such a well stabilized laser is known with 11 or 13 exact figures; it could reach 15 digits for clock standards, see below.

From Wikipedia till The end

Full width at half maximum (**FWHM**) is an expression of the extent of a function, given by the difference between the two extreme values of the independent variable at which the dependent variable is equal to half of its maximum value.

FWHM is applied to such phenomena as the duration of pulse waveforms and the spectral width of sources used for optical communications and the resolution of spectrometers.

The term *full duration at half maximum* (FDHM) is preferred when the independent variable is time.



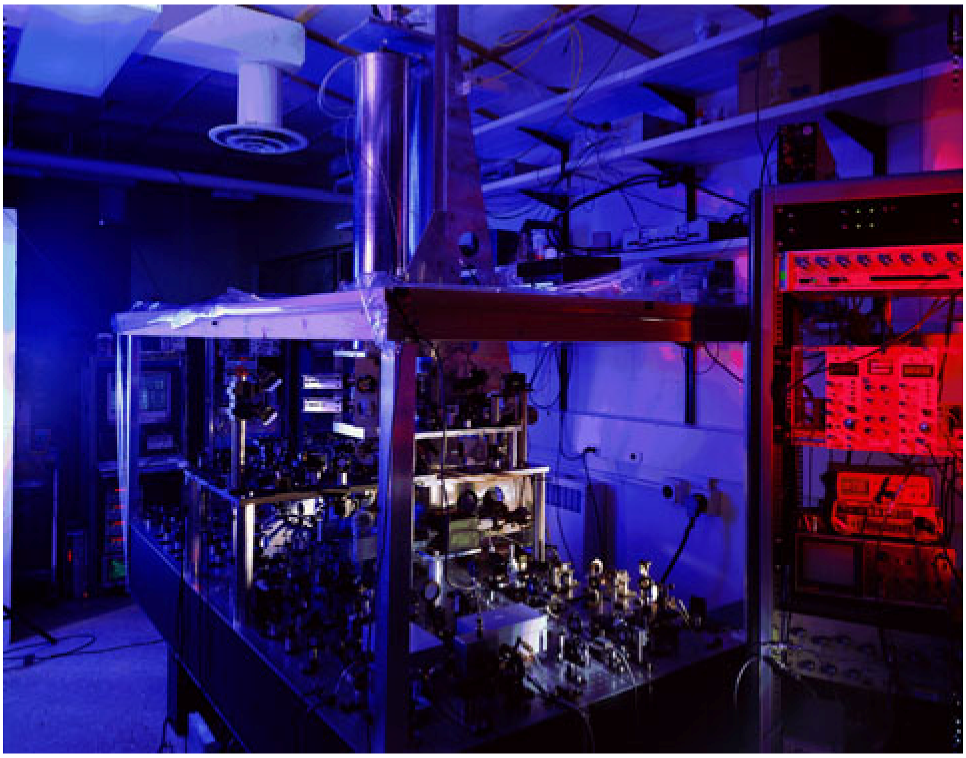
Full width at half maximum $\Delta x_{FWHM} = x_2 - x_1$

From Wikipedia, The evolution of the performances of clocks since 1950

NIST-F1 Cesium Fountain Atomic Clock

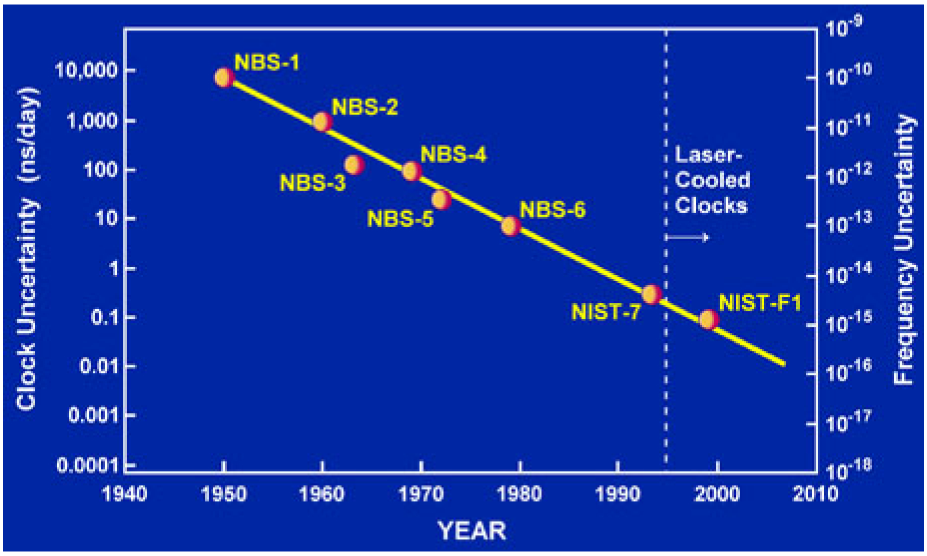
NIST [National Institute of Standards and Technology]

The Primary Time and Frequency Standard for the United States



NIST-F1, the nation's primary time and frequency standard, is a cesium fountain atomic clock developed at the NIST laboratories in Boulder, Colorado. NIST-F1 contributes to the international group of atomic clocks that define Coordinated Universal Time (UTC), the official world time. Because NIST-F1 is among the most accurate clocks in the world, it makes UTC more accurate than ever before.

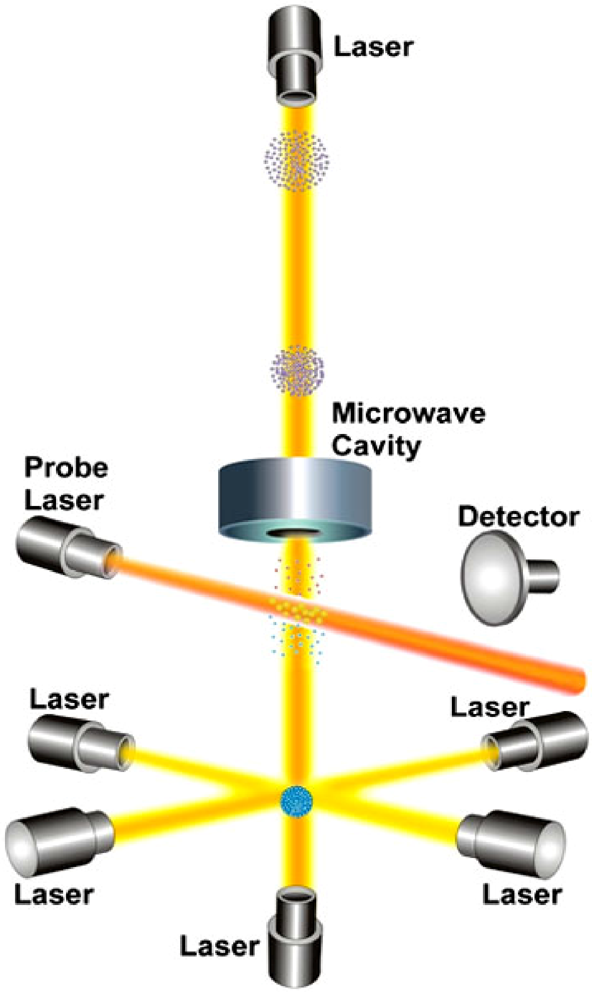
The uncertainty of NIST-F1 is continually improving. In 2000 the uncertainty was about 1×10^{-15}, but as of the summer of 2005, the uncertainty has been reduced to about 5×10^{-16}, which means it would neither gain nor lose a second in more than 60 million years! The graph below shows how NIST-F1 compares to [previous atomic clocks built by NIST](). It is now approximately ten times more accurate than NIST-7, a cesium beam atomic clock that served as the United State's primary time and frequency standard from 1993-1999.



Technical Description

NIST-F1 is referred to as a fountain clock because it uses a fountain-like movement of atoms to measure frequency and time interval. First, a gas of cesium atoms is introduced into the clock's vacuum chamber. Six infrared laser beams then are directed at right angles to each other at the center of the chamber. The lasers gently push the cesium atoms together into a ball. In the process of creating this ball, the lasers slow down the movement of the atoms and cool them to temperatures near absolute zero.

Two vertical lasers are used to gently toss the ball upward (the "fountain" action), and then all of the lasers are turned off. This little push is just enough to loft the ball about a meter high through a microwave-filled cavity. Under the influence of gravity, the ball then falls back down through the microwave cavity.



The round trip up and down through the microwave cavity lasts for about 1 second. During the trip, the atomic states of the atoms might or might not be altered as they interact with the microwave signal. When their trip is finished, another laser is pointed at the atoms. Those atoms whose atomic states were altered by the microwave signal emit light (a state known as fluorescence). The photons, or the tiny packets of light that they emit, are measured by a detector.

This process is repeated many times while the microwave signal in the cavity is tuned to different frequencies. Eventually, a microwave frequency is found that alters the states of most of the cesium atoms and maximizes their fluorescence. This frequency is the natural resonance frequency of the cesium atom (9,192,631,770 Hz), or the frequency used to define the second.

MPEG Video Demonstration of How a Cesium Fountain Works

The combination of laser cooling and the fountain design allows NIST-F1 to observe cesium atoms for longer periods, and thus achieve its unprecedented accuracy. Traditional cesium clocks measure room-temperature atoms moving at several hundred meters per second. Since the atoms are moving so fast, the observation time is limited to a few milliseconds. NIST-F1 uses a different approach. Laser cooling drops the temperature of the atoms to a few millionths of a degree above absolute zero, and reduces their thermal velocity to a few centimeters per second. The laser cooled atoms are launched vertically and pass twice through a microwave cavity, once on the way up and once on the way down. The result is an observation time of about one second, which is limited only by the force of gravity pulling the atoms to the ground.

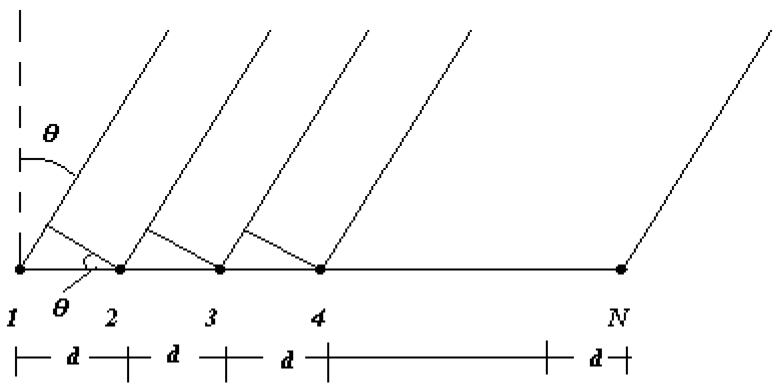
As you might guess, the longer observation times make it easier to tune the microwave frequency. The improved tuning of the microwave frequency leads to a better realization and control of the resonance frequency of cesium. And of course, the improved frequency control leads to what is one of the world's most accurate clocks.

Credits

NIST-F1 was developed by Steve Jefferts and Dawn Meekhof of the Time and Frequency Division of NIST's Physical Measurement Laboratory in Boulder, Colorado. It was constructed and tested in less than four years. The current NIST-F1 team includes physicists Steve Jefferts and Tom Heavner. The end

4. Interference of several harmonic plane waves

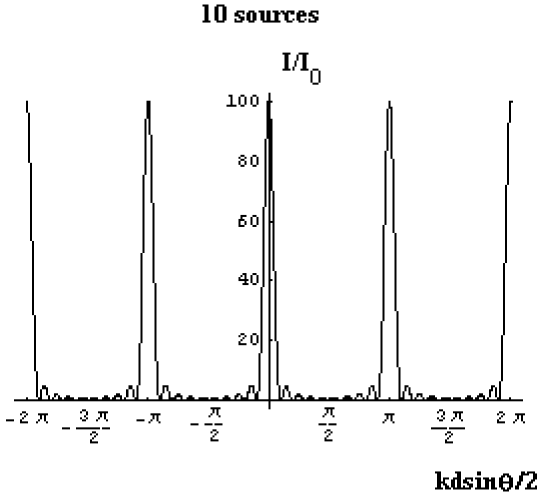
N perfectly coherent identical sources are evenly arranged in a straight line. The distance between two adjacent sources is *d*. We compute the intensity in a very far point, so far that we may assume the propagation directions are parallel. The propagation direction is given by angle θ, as in the figure. Each source has the intensity I_0.



$$E_{tot}=\sum_{j=1}^{N}E_j=E_0\{\exp[i(\omega t-kr)+\exp[i(\omega t-k(r+d\sin\theta)+\exp[i(\omega t-k(r+2d\sin\theta)+...$$

$$+\exp[i(\omega t-k(r+(N-1)d\sin\theta)\}=E_0\exp[i(\omega t-kr)]\frac{1-\exp[-iNkd\sin\theta]}{1-\exp[-ikd\sin\theta]}$$

$$I_{tot}=I_0\frac{\sin^2\left[\frac{Nkd\sin\theta}{2}\right]}{\sin^2\left[\frac{kd\sin\theta}{2}\right]} \qquad \text{(I10)}$$



Problems: What are the positions of the minima ? What are the values of the small maxima ?

5. Methods to get coherent beams

a). With coherent sources: lasers in optics, antennae in electronics.

b). Division of the wave-front: Young (see the lectures).

c). Division of the intensity: Michelson, reflection-refraction on slim slabs (see the lectures).

The most difficult condition to accomplish: the constant phase difference between the sources. How is it realized ?

6. The importance of interference

Interferential devices are the basis of many of the most accurate and sensitive measurements made in science. Their uniqueness originates in the phase dependence on the path difference, namely in the spatial term $\exp[i(\omega t - k\Delta r)]$: $k\Delta r = 2\pi \frac{n\Delta r}{\lambda_0}$. In the visible domain $\lambda \approx 5 \cdot 10^{-7}\,\text{m}$, so for a path difference of just one micrometer a 4π phase shift becomes visible. But we are able to easily measure phase shifts and fringe shifts corresponding to just $\pi / 100$, hence to measure $\Delta r \approx \lambda / 200 \approx 2.5\,\text{nm}$. With special care the resolution could diminish 10 times or even more. The use of such small path differences is rather impressive. Any physical quantity which can be linked to a path difference may be measured with high accuracy. What are these quantities? Length, time, wavelength, but also those which change the refractive index: temperature, chemical composition, electric and magnetic fields and many others.