3.3. Oscilatii atenuate si fortate

Ecuatia:

$$m\ddot{x} + \gamma\dot{x} + kx = F(t) \tag{3.3.1}$$

sau

$$\ddot{x} + \frac{2}{\tau}\dot{x} + \omega_0^2 x = \frac{F(t)}{m} \tag{3.3.1'}$$

Pp ca forta exterioara este armonica (frecventa unica)

$$F(t) = F_0 \exp[i\Omega t] \tag{3.3.2}$$

Cautam solutii particulare de forma

$$x(t) = a \exp[i\Omega t] \tag{3.3.3}$$

Aceasta se numeste **Solutie stationara**.

$$a(\Omega) = \frac{F_0}{m} \frac{1}{\omega_0^2 - \Omega^2 + i\frac{2\Omega}{\tau}} = |a(\Omega)| \exp[i\theta(\Omega)]] \tag{3.3.4}$$

Tema: Cu $F_0 = 1,\ m = 1,\ \omega_0 = 100$ in SI , variati Ω si τ si reprezentati $|a(\Omega)|$ si $\theta(\Omega)$.

Rezonanta la $\Omega \cong \omega_0$ pentru atenuari mici, dar poate diferi mult.

Studiu calitativ.

$$|a(\Omega)| = \frac{F_0}{m} \frac{1}{\sqrt{\left(\omega_0^2 - \Omega^2\right)^2 + \frac{4\Omega^2}{\tau^2}}} \tag{3.3.5}$$

$$\tan\theta(\Omega) = -\frac{2\Omega/\tau}{\omega_0^2 - \Omega^2} \tag{3.3.6}$$

Frecventa externa mica $\Omega \to 0$ $|a(0)| = \frac{F_0}{m}\frac{1}{\omega_0^2} = \frac{F_0}{k}$ $\theta(0) = 0$ (3.3.7)

Frecventa externa mare $\Omega \to \infty$ $|a(\infty)| \to 0$ $\theta(\infty) = \pi$ (3.3.8)

Frecventa $\Omega = \omega_0$

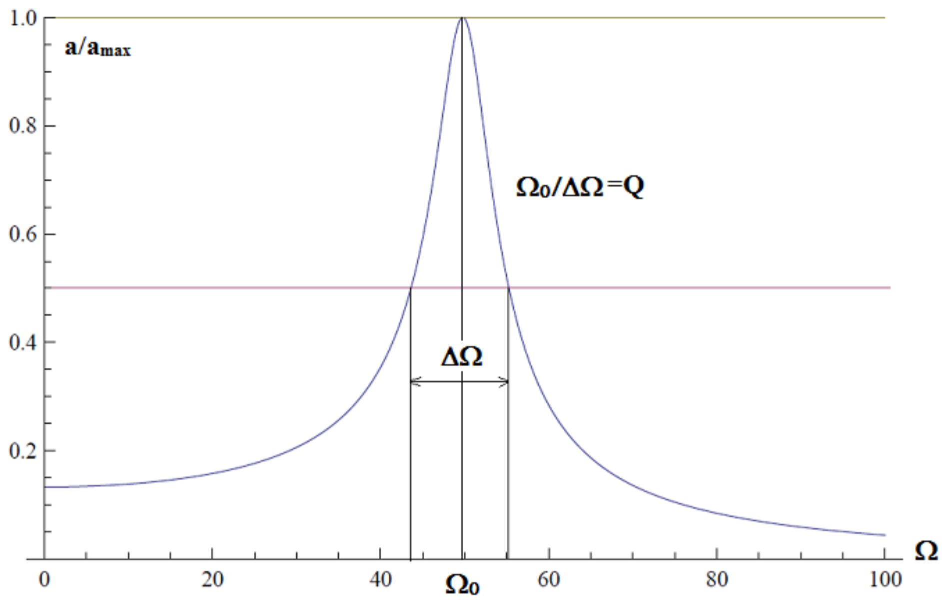
$$|a(\omega_0)| = \frac{F_0}{m}\frac{1}{2\,\omega_0/\tau} = \frac{F_0}{m\omega_0^2}\frac{1}{2/\omega_0\tau} = Q\frac{F_0}{m\omega_0^2} = Q|a(0)| \qquad (3.3.8)$$

$$\theta(\omega_0) = \pi/2 \qquad (3.3.9)$$

Factorul de calitate

Am introdus **factorul de calitate al oscilatorului *Q***:

$$Q = \frac{\omega_0 \tau}{2} \qquad (3.3.10)$$

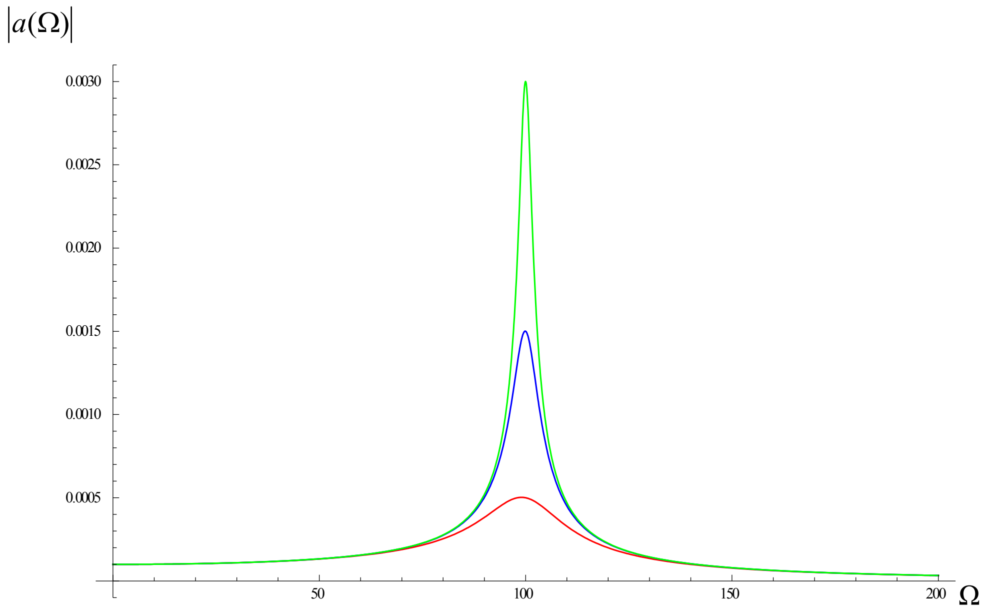


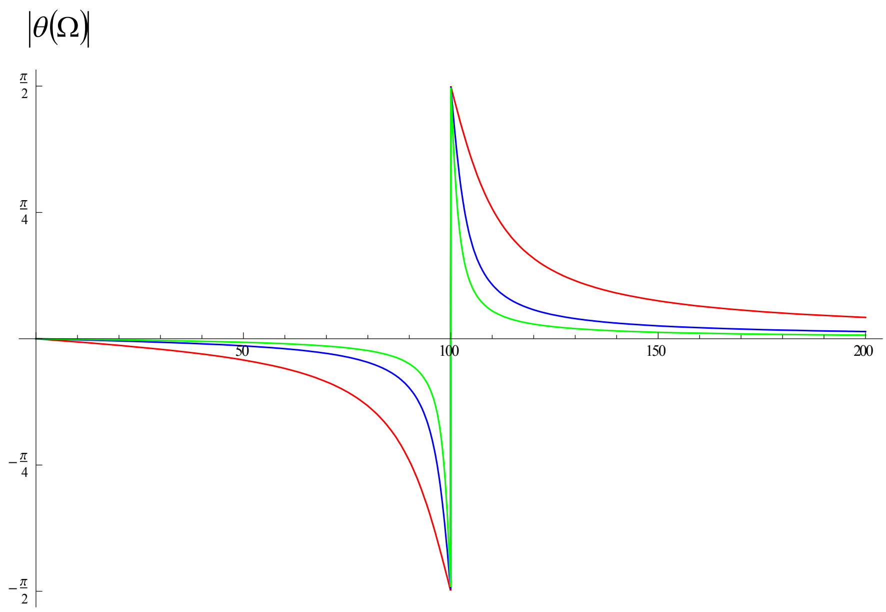
Cu o buna aproximatie

$$\frac{\omega_0}{\Delta\Omega} \cong Q \qquad (3.3.11)$$

$\Delta\Omega$ **se numeste** largime la semiinaltime (FWHM ***full width at half-maximum***). Se arata ca

$$\Omega_{\max} = \omega_0 \sqrt{1 - \frac{1}{2Q^2}} \qquad (3.3.12)$$



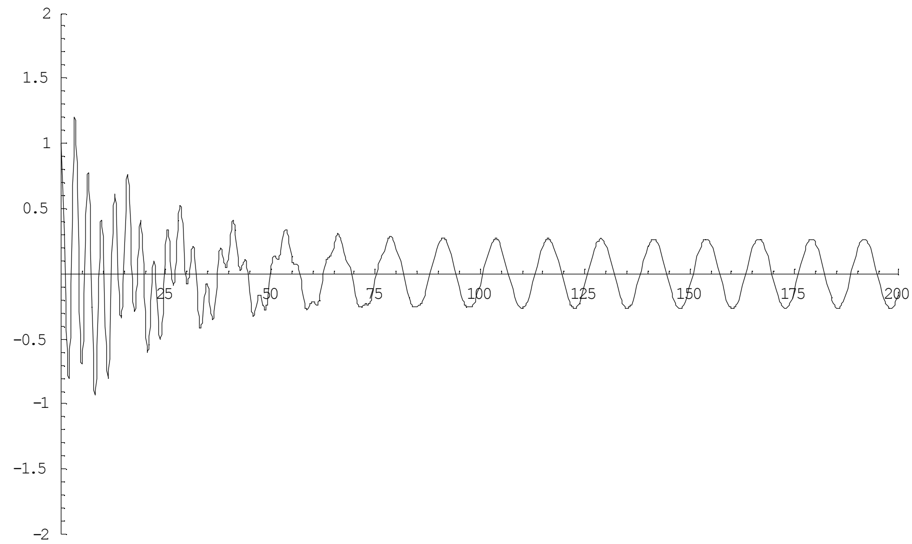


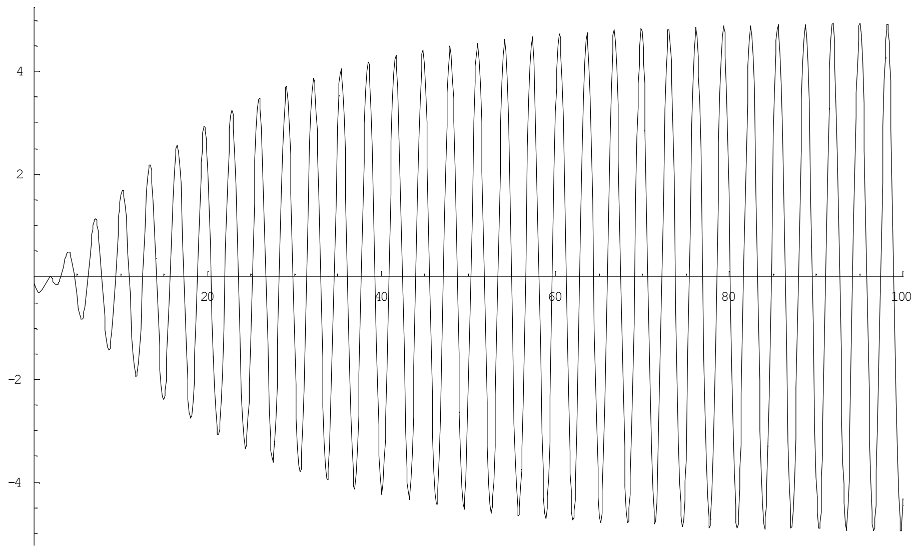
Solutia tranzitorie

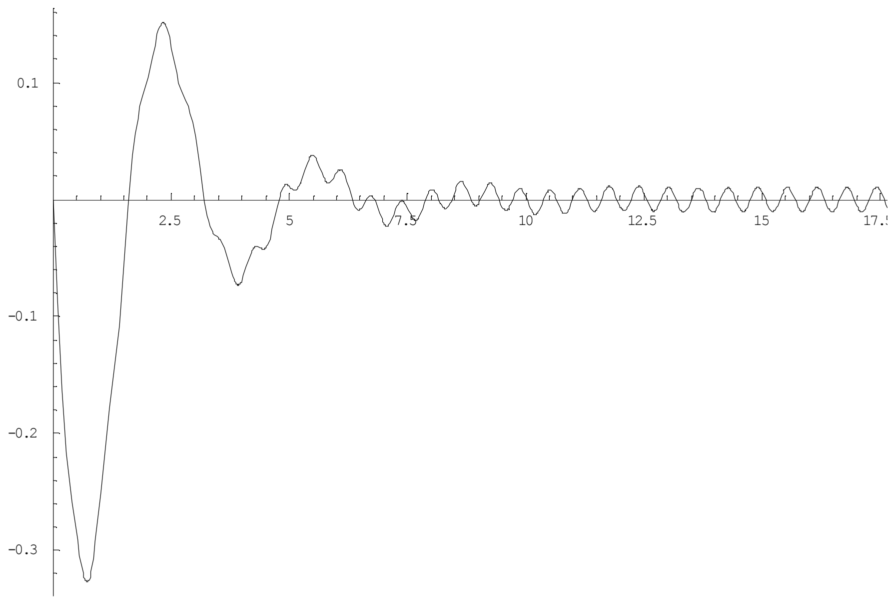
Se arata ca <u>*solutia generala a ecuatiei neomogene*</u> (3.3.1')

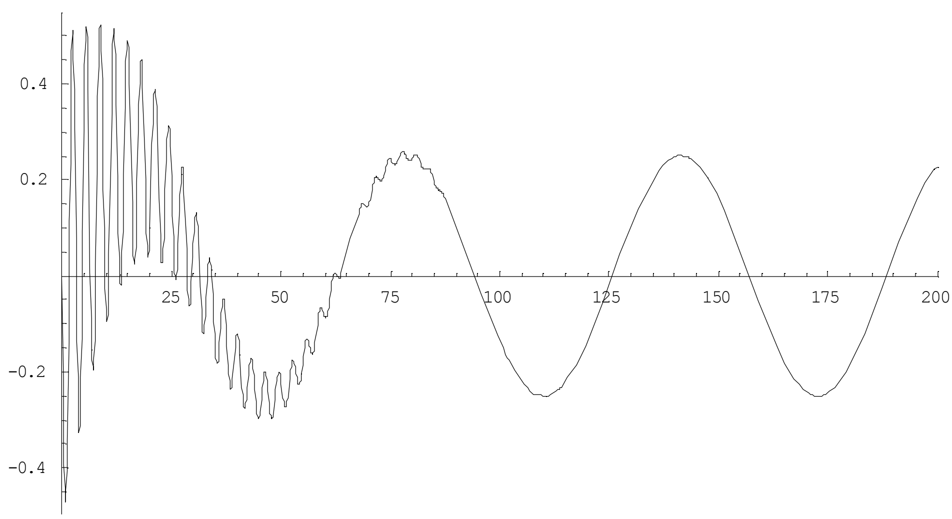
$$\ddot{x} + \frac{2}{\tau}\dot{x} + \omega_0^2 x = \frac{F(t)}{m}$$

Este suma dintre solutia generala a ecuatiei omogene ($F(t)$=0) si o solutie particulara a ecuatiei complete, de ex. (3.3.3) si (3.3.4). Iata cateva cazuri in care solutia stationara apare dupa catva timp si este clar delimitata de cea tranzitorie:



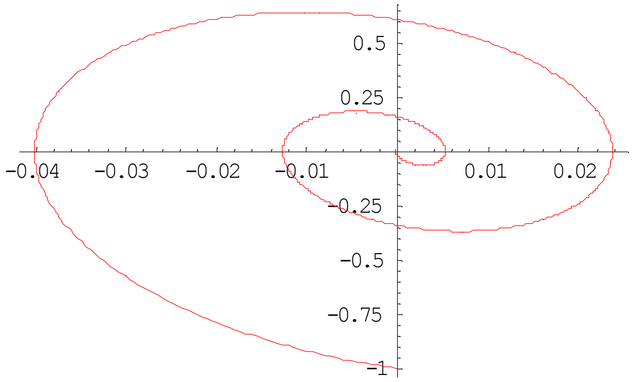




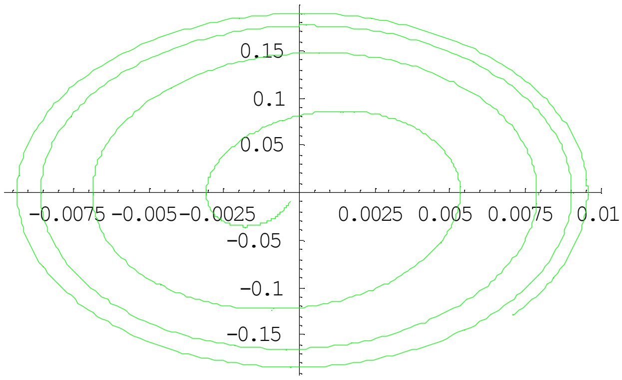


Pentru rezonanta $\Omega = \omega_0$ prezentam si traiectoria in spatiul fazelor

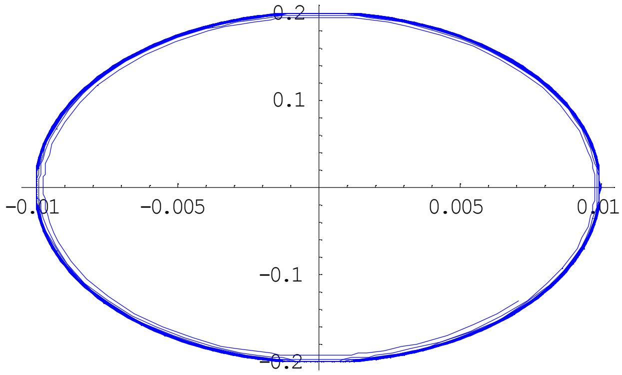
Rosu efectul atenuarii este mai mare decat cel al fortei exterioare, plecand din starea initiala $x(0)=0,\ \dot{x}(0)=-1$, oscilatorul se apropie de originea $x(0)=0,\ \dot{x}(0)=0$



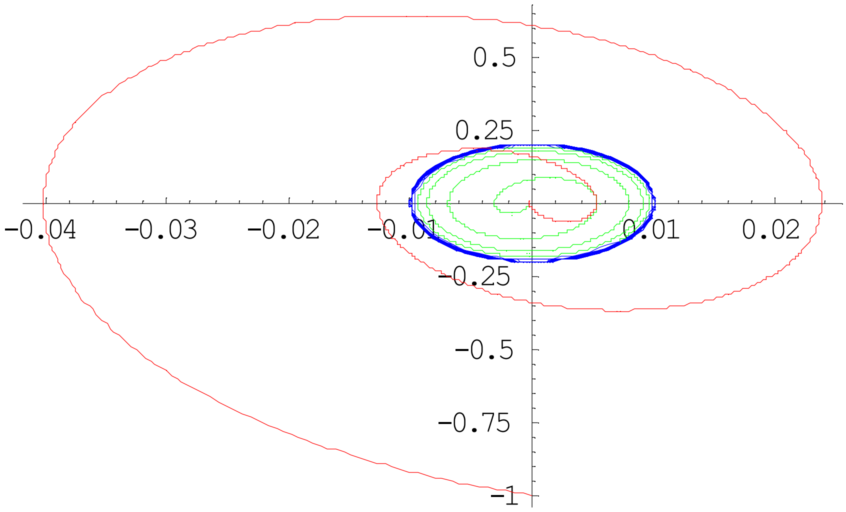
Verde Forta exterioara reporneste oscilatorul care se misca intr-un regim “mixt”,



Albastru Regiunea stationara



Toata miscarea



Daca nu e rezonanta, amplitudinea finala e mai mica decat cea initiala