3.2. Oscilatii atenuate 1D

De obicei exista frecare. Pentru simplitate presupunem ca frecarea este proportionala cu viteza corpului. Legea a II-a Newton se scrie: $m\ddot{x} = F_{el} + F_{fr} = -kx - \gamma\dot{x}$, sau

$$m\ddot{x} + \gamma\dot{x} + kx = 0 \qquad (3.2.1)$$

unde γ >0 este un factor de atenuare. Impartim cu *m*:

$$\ddot{x} + \frac{2}{\tau}\dot{x} + {\omega_0}^2 x = 0 \qquad (3.2.2)$$

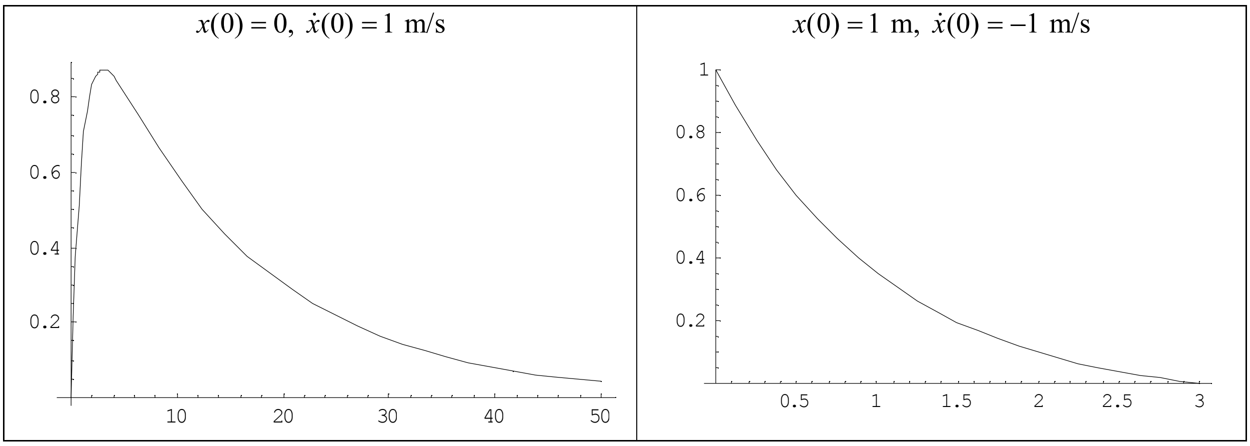
Am notat $\gamma/m = 2/\tau$. Ecuatia caracteristica $r^2 + \frac{2}{\tau}r + \omega_0^2 = 0$ are radacinile

$$r_{1,2} = -\frac{1}{\tau} \pm \sqrt{\left(\frac{1}{\tau}\right)^2 - \omega_0^2} \qquad (3.2.3)$$

Solutie analitica: $x(t) = e^{-t/\tau}\left(A\exp\left[\sqrt{\left(\frac{1}{\tau}\right) - \omega_0^2}\right] + B\exp\left[-\sqrt{\left(\frac{1}{\tau}\right) - \omega_0^2}\right]\right)$ (3.2.4)

a). $\frac{1}{\tau} > \omega_0$. **Radacini reale negative**. Miscare atenuata, eventual cu o crestere initiala care depinde de conditiile initiale:

Exemple: $\tau = 1s^{-1}$, $\omega_0 = 0.25s^{-1}$

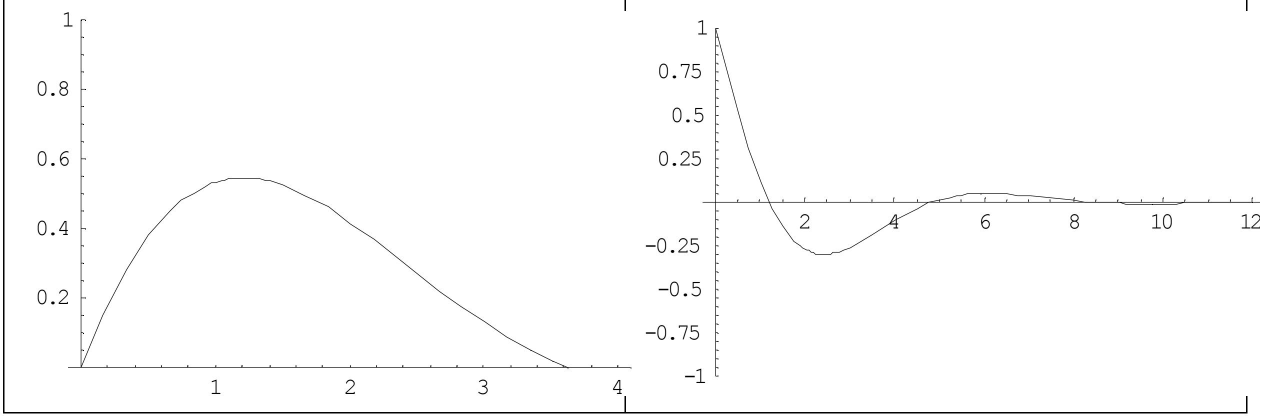


b). $\frac{1}{\tau} = \omega_0$. **Atenuare critica**. Radacini egale. Solutie analitica:

$$x(t) = (At + B)e^{-t/\tau} \tag{3.2.5}$$

Exemple

$x(0) = 0,\ \dot{x}(0) = 1$ m/s	$x(0) = 1$ m, $\dot{x}(0) = -1$ m/s

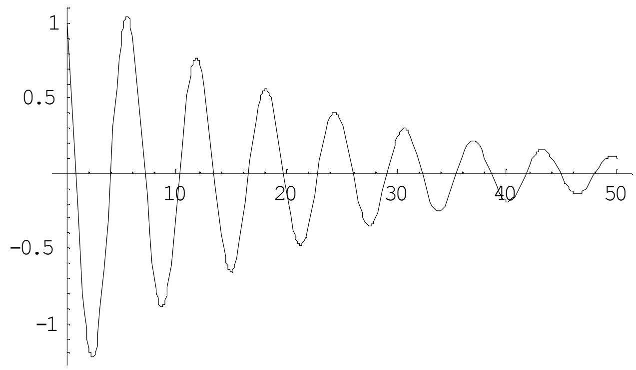


c) $\frac{1}{\tau} < \omega_0$. **Atenuare mica.** Radacini complex conjugate.

Solutie analitica: $x(t) = e^{-t/\tau}\left(A\sin(\omega t) + A\cos(\omega t)\right) = Ce^{-t/\tau}\sin(\omega t + \varphi_0)$ (3.2.6)

Frecventa unghiulara efectiva este

$$\omega = \sqrt{\omega_0^2 - \left(\frac{1}{\tau}\right)^2} \tag{3.2.7}$$



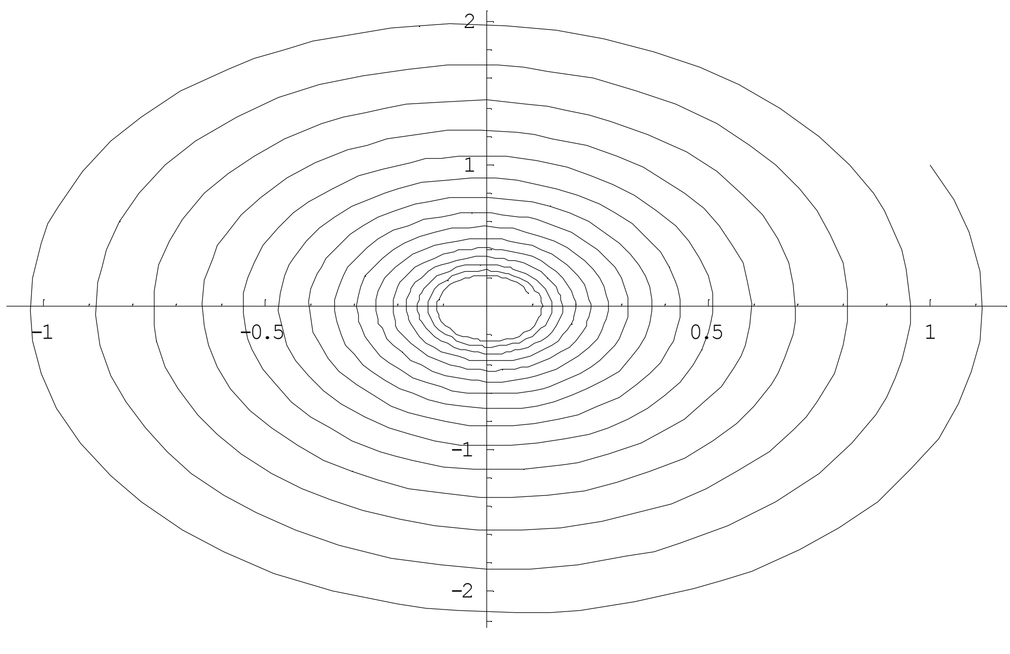
Este o oscilatie cu amplitudine descrescatoare si cuasi-perioada

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\omega_0^2 - \left(\frac{1}{\tau}\right)^2}} \tag{3.2.8}$$

Se defineste **decrementul logaritmic**

$$\delta = \ln \frac{x(t)}{x(t+T)} = \frac{T}{\tau} \tag{3.2.9}$$

In spatiul fazelor punctul reprezentativ parcurge o spirala care tinde spre origine:



Ce reprezinta o spirala care se departeaza de origine ?