Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Chaos, Solitons and Fractals xxx (2006) xxx–xxx
CHAOS SOLITONS & FRACTALS
www.elsevier.com/locate/chaos

Golden mean relevance for chaos inhibition in a system of two coupled modified van der Pol oscillators

Cristina Stan [a,*], C.P. Cristescu [a], M. Agop [b]

[a] *Department of Physics, "Politehnica" University of Bucharest, 313 Spl. Independentei, RO-060042, Bucharest, Romania*
[b] *Department of Physics, "Gh. Asachi" Technical University of Iassy, Blvd. Mangeron, 700029 Iassy, Romania*

Accepted 18 April 2006

Communicated by Prof. L.M. Cvnjac

Abstract

In this work, we present a novel evidence of the importance of the golden mean criticality of a system of oscillators in agreement with El Naschie's E-infinity theory. We focus on chaos inhibition in a system of two coupled modified van der Pol oscillators. Depending on the coupling between the two oscillators, the system shows chaotic behavior for different ranges of the coupling parameter. Chaos suppression, as a transition from irregular behavior to a periodical one, is induced by perturbing the system with a harmonic signal with amplitude considerably lower than the value which causes entrainment. The frequency of the perturbation is related to the main frequencies in the spectrum of the freely running system (without perturbation) by the golden mean. We demonstrate that this effect is also obtained for a perturbation with frequency such that the ratio of half the frequency of the first main component in the freely running chaotic spectrum over the frequency of the perturbation is very close (five digits coincidence) to the golden mean. This result is shown to hold for arbitrary values of the coupling parameter in the various ranges of chaotic dynamics of the free running system.

1. Introduction

Control of chaos and synchronization phenomena in coupled nonlinear systems have been extensively studied in various interdisciplinary fields. Numerical simulation and experimental methods of chaos control and synchronization were reported in physical systems with major impact on many aspects of science and engineering [1–6]. The behavior of coupled nonlinear oscillators consists in a diversity of dynamical phenomena when the nonlinearity or the coupling strength is increased. Since the case of two oscillators is a prototype model for understanding the phenomena in more complicated systems consisting of a large number of coupled oscillators, many theoretical as well as experimental studies have been dedicated to this model [7–9].

* Corresponding author.
E-mail address: cstan@physics.pub.ro (C. Stan).

0960-0779/$ - see front matter © 2006 Published by Elsevier Ltd.
doi:10.1016/j.chaos.2006.04.056

Generally, there are various ways to control chaos, the main ones being feedback control and weak perturbations. Feedback control methods are used to control chaos by stabilizing a desired unstable periodic orbit which is embedded in a chaotic attractor [1,2,7,8], while no feedback methods suppress chaotic behavior by applying a weak perturbation to some control parameter or certain variables of the system [3–6,10,12].

We study chaos inhibition induced by an external periodic perturbation in the nonlinear dynamics of coupled, identical, modified van der Pol oscillators. This system was previously used to model the dynamical behavior of an experimental device consisting of two electrical discharges running in the same glass tube [11].

We demonstrate that, for a particular frequency of a harmonic perturbation, suppression of chaos can be achieved. In the ensued regular dynamics, with increasing the amplitude of the periodic perturbation, the behavior is changed into a sequence of bifurcations.

The particular relevance of the golden mean for the El Naschie's relationship between the frequencies of a simple two degrees of freedom linear vibration consisting of two gravitational pendulums of unity masses connected by linear elastic springs is given simply by [13–16]:

$$\omega_1 = \phi = \frac{\sqrt{5}-1}{2}, \quad \omega_2 = \frac{1}{\phi} = \frac{\sqrt{5}+1}{2}. \tag{1}$$

El Naschie has shown recently that the golden mean and the geometry of Moebius modular group are closely connected in a similar way to the connection between the geometry of a classical diffusion on a fractal lattice to that of a Schrödinger equation [16]. This is particularly clear as $1/\phi$ is one of the two distinct fixed points of Moebius modular transformation.

In this paper, we demonstrate by numerical simulation that chaotic dynamics in the free running system can be inhibited by a small sinusoidal perturbation provided that the relationship between the frequency of the perturbation and some main characteristic frequency of the unperturbed system is in close relationship to the golden mean.

2. Mathematical model and results

The mathematical model consists of two bi-directionally coupled modified van der Pol oscillators. This system of equations was found to be a reasonable computational model for the current oscillations in the inter-anode space of a twin electrical discharge plasma [12].

The dynamics of this model is considerably richer than observed in our experimental system; however we shall focus only on a particular response of the model system to an externally applied periodical perturbation.

The computational system consists of the following equations:

$$\begin{aligned}
\frac{dx_1}{dt} &= x_2 + m x_4, \\
\frac{dx_2}{dt} &= -c(x_1^2 - 1)x_2 - x_1 + m(x_4 + x_3) + e\cos x_5, \\
\frac{dx_3}{dt} &= x_4 - m x_2, \\
\frac{dx_4}{dt} &= -f(x_3^2 - 1)x_4 - x_3 - m(x_2 + x_1) - e\cos x_5, \\
\frac{dx_5}{dt} &= g.
\end{aligned} \tag{2}$$

The fifth equation is introduced in order to transform the system into an autonomous one.

Without any perturbation ($e = 0$), by changing the coupling parameter m, considered as control parameter, different oscillatory and chaotic regimes can be observed.

For a fixed value of the parameter m, control of the specific behavior can be obtained using an external perturbation. Here, we only consider a particular range of amplitudes thereof considerably lower than the value corresponding to entrainment of the system that can change the chaotic dynamics into a regular one.

In the following, we give two values for the perturbation frequency g: the first one is the value used in the integration of the system (2) and the second one, expressed in arbitrary units represents the corresponding value in the FFT spectrum. The computation is carried out for the numerical values $f = c = 1$.

We use the FFT spectrum to carefully measure the frequency of the first main component of the chaotic spectrum f_1 and of the perturbation g. We observe that in order to obtain chaos suppression, for $f_1 = 213.063$ (arb. units) the perturbation frequency has to be $g = 131.682$ (arb. units). Their ratio is:

$$\frac{f_1}{g} = \frac{213.063}{131.682} = 1.61801. \tag{3}$$

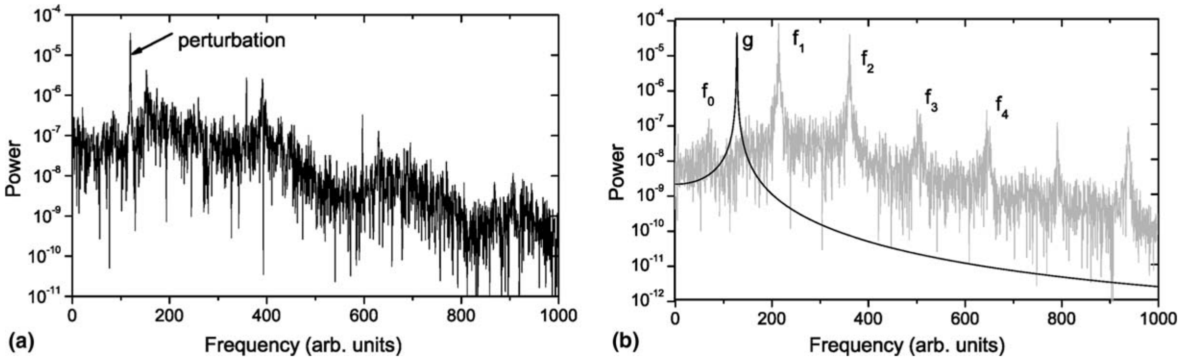

Fig. 1. (a) FFT spectrum for the system in the presence of a harmonic perturbation with frequency $g = 0.75$ (120.153 arb. units) slightly below the value $g = 0.8$ (131.682 arb. units) given by Eq. (3) showing no regular dynamics and (b) FFT power spectrum for the unperturbed system (grey), showing chaotic dynamics, and the spectrum of the harmonic perturbation (black) leading to chaos suppression. The value of the coupling parameter is $m = 1.53$ for both spectra.

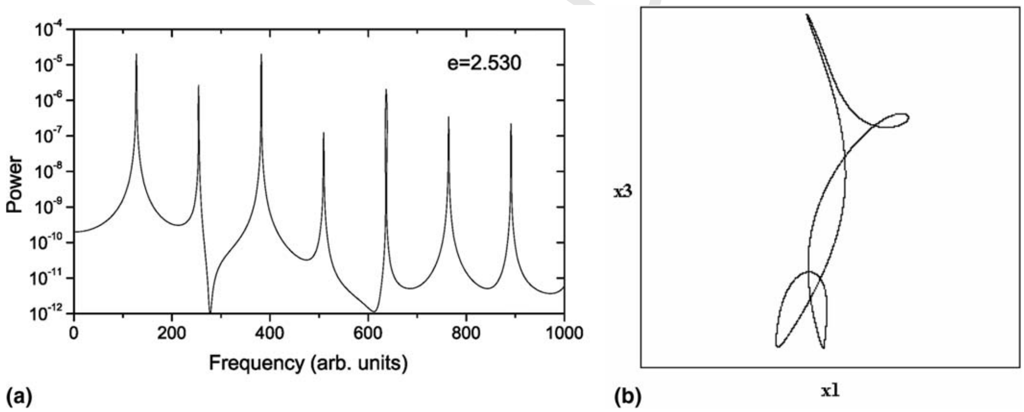

Fig. 2. (a) FFT power spectrum of the perturbed system for the value of the coupling parameters $m = 1.53$, showing regular dynamics and (b) the corresponding (x_1, x_3) phase space portrait.

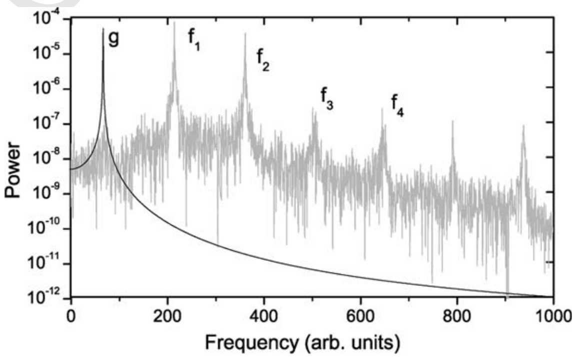

Fig. 3. FFT power spectrum for the unperturbed system (grey) for the value of the coupling parameter $m = 1.53$ showing chaotic dynamics and the spectrum of the harmonic perturbation (black) that corresponds to chaos suppression.

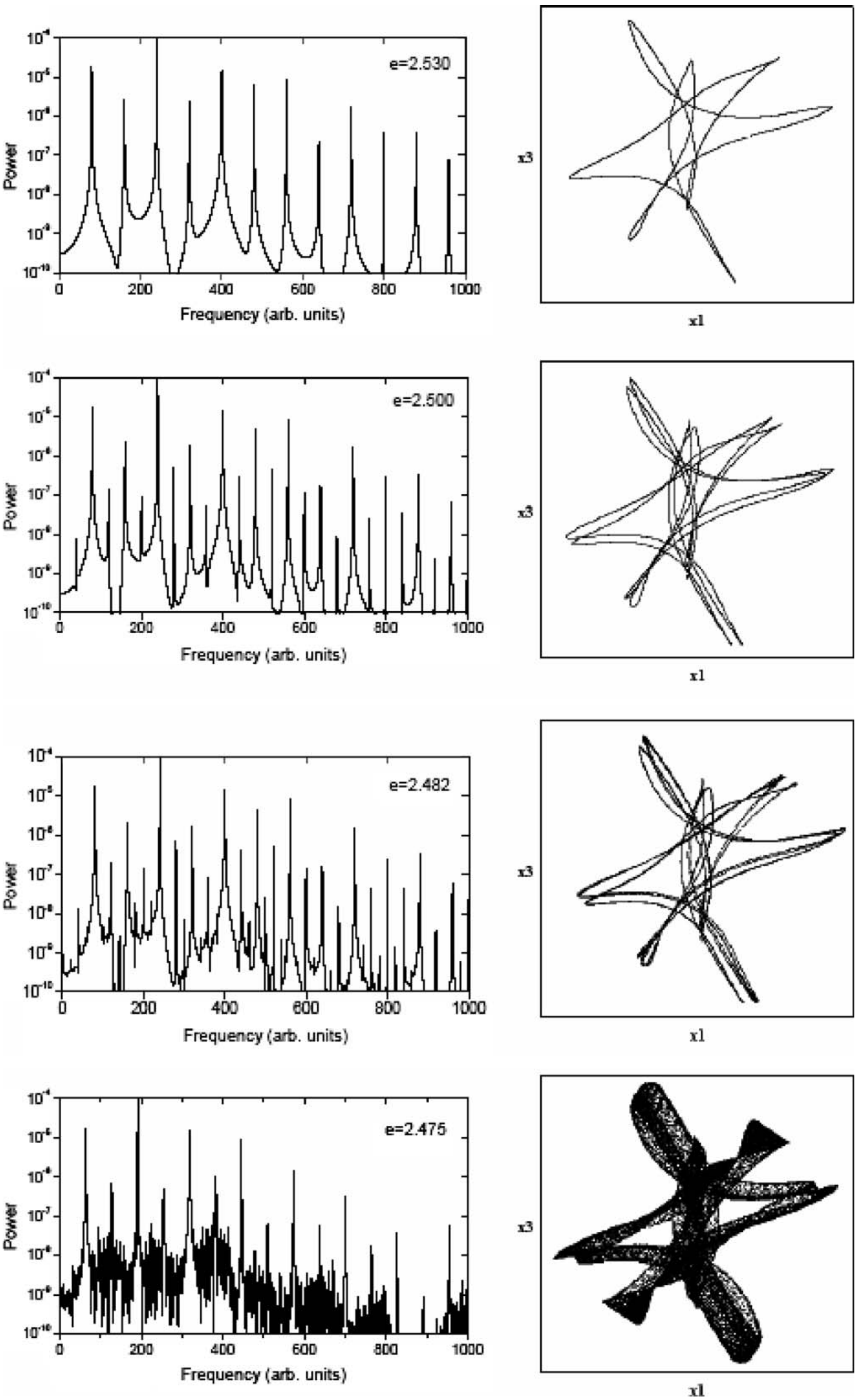

Fig. 4. FFT power spectra (left) and the corresponding phase portraits (right) for the values of the perturbation amplitude shown on each spectrum.

Comparing this number with the golden mean $1/\phi$, we find a five digits coincidence.

In Fig. 1a, the FFT spectrum for the system in the presence of a harmonic perturbation with frequency $g = 0.75$ (120.153 arb. units) slightly below the value $g = 0.8$ (131.682 arb. units) given by Eq. (3) is presented. There is no sign of chaos suppression.

In Fig. 1b, superimposed are shown the FFT power spectrum for the unperturbed system (grey), demonstrating chaotic dynamics, and the spectrum of the harmonic perturbation (black) for $g = 0.8$ (131.682 arb. units), that induces chaos suppression. For both situations in Fig. 1, the value of the coupling parameters is $m = 1.53$.

For the particular value of the perturbation frequency given by Eq. (3), chaos inhibition is obtained.

Fig. 2a shows the FFT power spectrum for the system in the presence of a harmonic perturbation with frequency $g = 0.8$ (131.682 arb. units) and amplitude $e = 2.53$, for the value of the coupling parameter $m = 1.53$. Side by side with the power spectrum, the corresponding (x_1, x_3) phase space portrait is presented in Fig. 2b.

From Fig. 2, we notice that, for a slight variation in the perturbation frequency leading to the value given by Eq. (3), the chaotic dynamics characterized by the grey spectrum in Fig. 1b is changed into regular dynamics.

Chaos suppression is also observed for different frequencies of the harmonic perturbation given by the same equation (3), with f_1 replaced by f_2, f_3, f_4, etc. (see Fig. 1b). For each situation a different phase portrait (x_1, x_3) is obtained.

Another interesting result was obtained using a perturbation with frequency given by the relationship

$$\frac{\frac{1}{2}f_1}{g} = 1.6180. \quad (4)$$

We remark that in the spectrum of the unperturbed system, as can be observed from Fig. 3 (the grey spectrum) or Fig. 1b (the grey spectrum), no distinct component is present at the frequency $\frac{1}{2}f_1$; no relationship seems to exist between f_0 and $\frac{1}{2}f_1$.

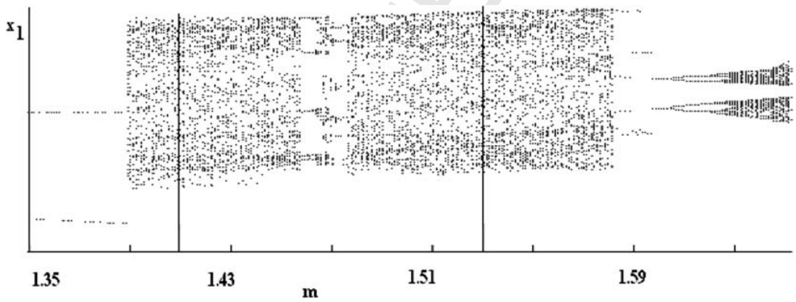

Fig. 5. Bifurcation diagram of x_1 using as control parameter the coupling constant m. the vertical lines show the values used in this study ($m = 1.53$ and $m = 1.41$).

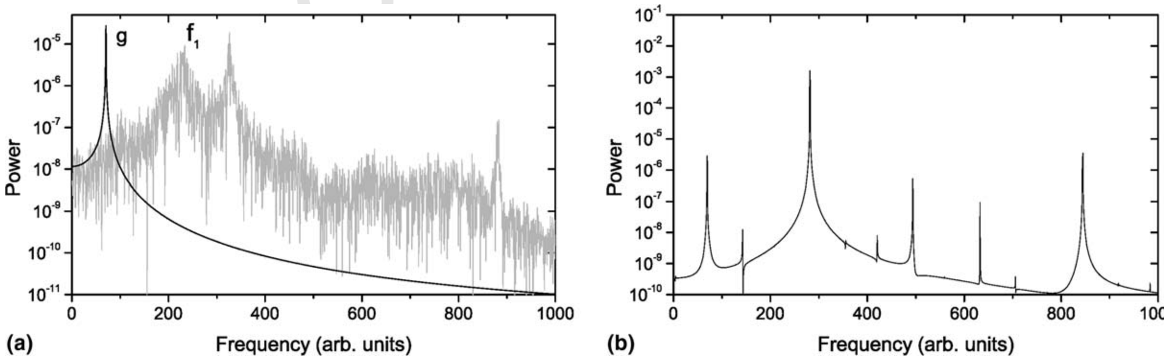

Fig. 6. (a) FFT power spectrum for the unperturbed system (grey) for the value of the coupling parameter $m = 1.41$, showing chaotic dynamics and the spectrum of the harmonic perturbation (black) corresponding to the chaos suppression shown by the spectrum in (b) and (b) FFT spectrum of regular dynamics for the perturbation with frequency $g = 71.107$ (arb. units).

In conditions of chaos suppression achieved by a perturbation with the frequency given by Eq. (4), we study the change in the dynamics of system (2) caused by the variation of the amplitude of the perturbation. A sequence of period doubling bifurcations obtained for this frequency is shown in Fig. 4. From top to bottom, FFT spectra are shown in the left for the values of the perturbation amplitude shown on each spectrum and in the right the corresponding phase portraits are presented.

In order to check the validity of the golden mean condition as shown by Eq. (4) on our system, we considered another value of the coupling constant m in a different region of the chaotic dynamics of the unperturbed system. A fragment of the bifurcation diagram of the free running system, using the coupling constant (m) as control parameter is shown in Fig. 5. The numerical values of the other parameters are as before: $f = c = 1$.

The two values of m in our numerical simulation are marked by the vertical lines on the bifurcation diagram shown in Fig. 5.

Fig. 6a shows the FFT power spectrum for the unperturbed system (grey) for an arbitrary value of the coupling parameter in the first range of chaotic dynamics in Fig. 5 ($m = 1.41$); superimposed is the spectrum of the harmonic perturbation (black) that corresponds to chaos suppression as shown by the spectrum in Fig. 6b.

Chaos suppression is obtained for the frequency of the perturbation g in a relationship to the first main component of the chaotic spectrum f_1 given by Eq. (4). The specific values are $f_1 = 230.103$ (arb. units) and $g = 71.107$ (arb. units).

3. Conclusions

In this work, we present an important aspect for the golden mean criticality of systems of oscillators as pointed out by El Naschie. We study chaos inhibition in a system of two coupled modified van der Pol oscillators. Here, by suppression of chaos we mean the change from irregular behavior to a periodic one induced by a harmonic perturbation with a frequency related to the main frequencies in the spectrum of the free running system by the golden mean. We consider a range of amplitudes of the perturbation lower than the value corresponding to entrainment of the system.

Also, we demonstrate that the same effect is obtained for a perturbation frequency such that the ratio of half the frequency of the first main component in the free running spectrum (chaotic) over the frequency of the perturbation is very close (five digits coincidence) to the golden mean. This result is shown to hold for arbitrary values of the coupling parameter in the ranges of chaotic dynamics of the unperturbed system.

References

[1] Agiza HN. Controlling chaos for the dynamical system of coupled dynamics. Chaos, Solitons & Fractals 2002;13:341–52.
[2] Harb AM, Jabbar NA. Controlling Hopf bifurcation and chaos in a small power system. Chaos, Solitons & Fractals 2003;18:1055–63.
[3] Belhaq M, Houssni M. Suppression of chaos in averaged oscillator driven by external and parametric excitations. Chaos, Solitons & Fractals 2000;11:1237–46.
[4] Kapitaniak T. Continuous control and synchronization in chaotic systems. Chaos, Solitons & Fractals 1995;6:237–44.
[5] Lynch S, Steele AL. Controlling chaos in nonlinear optical resonators. Chaos, Solitons & Fractals 2000;11:721–8.
[6] Agop M, Griga V, Ciobanu B, Buzea C, Stan C, Tatomir D, et al. The uncertainty relation for anassembly of Planck type oscillators. A possible GR-quantum mechanics connection. Chaos, Solitons & Fractals 1997;5:809–21.
[7] Ott E, Grebogi N, Yorke J. Controlling chaos. Phys Rev Lett 1990;64(11):1196–9.
[8] Cristescu CP, Stan C, Alexandroaei D. Control of chaos by random noise in a system of two coupled perturbed van der Pol oscillators modeling an electrical discharge plasma. Phys Rev E 2002;66:016602–8.
[9] Deng Bin, Wang Jiang, Fei Xiangyang. Synchronizing two coupled chaotic neurons in external electrical stimulation using backstepping control. Chaos, Solitons & Fractals 2006;29:182–9.
[10] Cristescu CP, Stan C, Alexandroaei D. Dynamic control by sinusoidal perturbation and by Gaussian noise of a system of two nonlinear oscillators: computation and experimental results. Phys Rev E 2004;70:016613–8.
[11] Stan C, Cristescu CP, Alexandroaei D. Modelling of the temporal behaviour of the inter-anode current in a system of two coupled electrical discharges. Contrib Plasma Phys 2002;42:81–7.
[12] Stan C, Cristescu CP, Alexandroaei D. New investigations on the dynamics of externally perturbed states of a chaotic discharge plasma double layer. J Optoelectron Adv Mater 2005;7(5):2451–4.
[13] El Naschie MS. A review of the ε^{∞} theory and the mass spectrum of the high energy particles physics. Chaos, Solitons & Fractals 2004;19:209–36.
[14] El Naschie MS. On the exact mass spectrum of quarks. Chaos, Solitons & Fractals 2002;14:369–76.
[15] El Naschie MS. Modular groups in Cantorian ε^{∞} high energy physics. Chaos, Solitons & Fractals 2002;16:353–72.
[16] El Naschie MS. On a class of general theories for high energy particle physics. Chaos, Solitons & Fractals 2002;14:649–68.