Gravitomagnetic field, spontaneous symmetry breaking and a periodical property of space

M. Agop (1)(*), B. Ciobanu (1), C. Gh. Buzea (2), N. Rezlescu (2)
T. Horgoş (3) and C. Stan (4)

(1) *Department of Physics, Technical "Gh. Asachi" University - Iaşi 6600, România*
(2) *Superconductivity Research Laboratory, Institute of Technical Physics - Iaşi 6600, România*
(3) *Electrical Institute Research - Bistriţa, Năsăud, România*
(4) *Faculty of Physics, "Al. I. Cuza" University - Iaşi 6600, România*

(ricevuto il 14 Novembre 1996; approvato il 3 Giugno 1997)

Summary. — The connection between the absence of the Meissner gravitational effect and spontaneous symmetry breaking, the gravitational fluxon, the motion of a particle in a homogeneous gravitomagnetic field and the quantum gravitational Hall effect are studied.

PACS 04.20.Cv – Fundamental problems and general formalism.
PACS 04.90 – Other topics in general relativity and gravitation.

1. – Introduction

Recent studies have evidenced that, in the approximation of the weak fields and slow speeds ($v \ll c$) Einstein's equations are reduced to the Maxwell-type equations [1-3]:

$$\nabla \times \mathbf{B}_g = -4\pi \mathbf{j}_m + \frac{\partial \mathbf{E}_g}{\partial t}, \tag{1}$$

$$\nabla \times \mathbf{E}_g = -\frac{\partial \mathbf{B}_g}{\partial t}, \tag{2}$$

$$\nabla \cdot \mathbf{E}_g = -4\pi \varrho, \tag{3}$$

$$\nabla \cdot \mathbf{B}_g = 0, \tag{4}$$

$$\mathbf{B}_g = \nabla \times \mathbf{A}_g, \qquad \mathbf{E}_g = -\frac{\partial \mathbf{A}_g}{\partial t} - \nabla V_g, \tag{5}$$

(*) E-mail: picardout@phys-iasi.ro

where $\mathbf{E}_g$ is the gravitoelectric field, $\mathbf{B}_g$ is the gravitomagnetic field, $\mathbf{A}_g$ is the gravitomagnetic potential, V_g is the scalar potential, $j^i = \varrho v$ the density of the mass current, $\varrho = n_0 m$ is the proper mass density and n_0 is the number density of particles of mass m.

Since, in this approximation, the mass is the unique source of the gravitational field, we have

$$\begin{cases} T^{\alpha\beta} = \varrho u^\alpha u^\beta, \quad T^{00} = \varrho c^2, \quad T^{0i} = \varrho v^i = j^i, \quad T^{ij} = 0\,, \\ T^\alpha = T^{\alpha 0} = (\varrho, j^i), \quad \alpha, \beta = \overline{1}, \overline{4}, \quad i, j = \overline{1}, \overline{3}\,, \end{cases} \tag{6}$$

the equation of continuity

$$\frac{\partial \varrho}{\partial t} + \partial \cdot \mathbf{j} = 0\,, \tag{7}$$

and the equation of motion are geodesics,

$$u^\beta \nabla_\beta u^\alpha = \frac{\mathrm{d}^2 x^\alpha}{\mathrm{d}l^2} + \begin{Bmatrix} \alpha \\ \beta\gamma \end{Bmatrix} \frac{\mathrm{d}x^\beta}{\mathrm{d}l} \frac{\mathrm{d}x^\gamma}{\mathrm{d}l} = 0\,. \tag{8}$$

Equation (8) is analogous to the equation for the Lorentz force acting on an electrically charged particle, that is, (8) is of the form [1-3]

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \mathbf{E}_g + \mathbf{v} \times \mathbf{B}_g\,. \tag{9}$$

Generally, the connection of the electromagnetic-linear gravitational field operates almost perfectly [2, 3]. The only exception is—for the time being—the absence of the Meissner effect in the case of the linear gravitational field. The absence of such effect has been interpreted, in [4], as a "space crystallization".

The present paper is devoted to the following aspects: the connection between the absence of the Meissner effect and the spontaneous symmetry breaking; the gravitational fluxon, the motion of a microparticle in a homogeneous gravitomagnetic field, and the gravitational Hall quantum effect.

2. – The connection between the absence of the gravitational Meissner effect and the spontaneous symmetry breaking

For a particle with mass m, which is moving in a gravitational field ($\mathbf{A}_g$, V_g), the Lagrangian takes the form

$$L \approx \frac{1}{2} m v^2 + 4 m \mathbf{v} \mathbf{A}_g - m V_g \tag{10}$$

from which the canonical momentum P is defined via the Lagrangian (10) as

$$\mathbf{P} = \frac{\partial L}{\partial \mathbf{v}} = m\mathbf{v} + 4 m \mathbf{A}_g\,. \tag{11}$$

Since [5]

$$\mathbf{P} = \nabla S\,, \tag{12}$$

where S is called action, by multiplying (11) by n_0, one may obtain

$$n_0 \nabla S = \mathbf{j} + 4\varrho \mathbf{A}_{\mathrm{g}}. \tag{13}$$

By applying the curl operator to (12) and assuming that $n_0 = \mathrm{const}$, we get

$$\nabla \times \mathbf{j} = -\,4\varrho \mathbf{B}_{\mathrm{g}}. \tag{14}$$

We make the observation that (12) coincides with Nelson's relation [6], $\mathbf{P} = \hbar\nabla\theta$, if we admit the identity $S = \hbar\theta$, with θ a phase and $\hbar$ the reduced Planck constant. From (1), using the restriction $\partial \mathbf{E}_{\mathrm{g}} / \partial t = 0$, there results

$$\nabla \times \nabla \times \mathbf{B}_{\mathrm{g}} = \nabla(\nabla \cdot \mathbf{B}_{\mathrm{g}}) - \Delta \mathbf{B}_{\mathrm{g}} = -4\pi \nabla \times \mathbf{j} \tag{15}$$

and using (4) and (14), one may finally obtain

$$\Delta \mathbf{B}_{\mathrm{g}} + 16\pi\varrho \mathbf{B}_{\mathrm{g}} = 0\,. \tag{16}$$

Such a result evidences the space's periodicity property, which agrees with [4].

Further on, let us investigate to which extent this property of "crystallizing" the space may be correlated with the spontaneous symmetry breaking. One should therefore consider the Schrödinger-type Lagrangian

$$L = \frac{1}{2}(\nabla\Psi)^* \nabla\Psi - \frac{\beta}{4}\left(|\Psi|^2 + \frac{\alpha}{\beta}\right)^2 - \frac{1}{2}\mathbf{B}_{\mathrm{g}}^2, \tag{17}$$

where $\nabla_l = \partial_l - i(4m/\hbar)\,A_{gl}$, $l = \overline{1}, \overline{3}$ represents the covariant derivative [3, 7], Ψ is the wave function coupled minimally to the gravitomagnetic field, $4m/\hbar$ is the coupling constant, α is a "massic" term and $\beta > 0$ is the self-interaction constant.

Using the EL equations for the Lagrange function (17), the field equations are

$$-\nabla \times \mathbf{B}_{\mathrm{g}} = \frac{2im}{\hbar}(\Psi^* \nabla\Psi - \Psi\nabla\Psi^*) + \frac{16m^2}{\hbar^2}\mathbf{A}_{\mathrm{g}}\Psi^*\Psi\,, \tag{18}$$

$$\nabla_l \nabla_l \Psi = -\,\beta\Psi\left(|\Psi|^2 + \frac{\alpha}{\beta}\right). \tag{19}$$

In the case of spontaneous symmetry breaking, $\alpha < 0$ and $\Psi = \sqrt{-\alpha/\beta}$ [8], so that (18) becomes

$$\nabla \times \mathbf{B}_{\mathrm{g}} = -4\pi\mathbf{j} = -\,\frac{16m^2}{\hbar^2}\,\frac{\alpha}{\beta}\mathbf{A}_{\mathrm{g}}. \tag{20}$$

By utilization of the curl operator and considering (4), one may obtain from (20)

$$\Delta \mathbf{B}_{\mathrm{g}} - \frac{16m^2}{\hbar^2}\,\frac{\alpha}{\beta}\mathbf{B}_{\mathrm{g}} = 0\,. \tag{21}$$

Such a result indicates that spontaneous symmetry breaking implies a gravitational Meissner effect with a penetration depth

$$\lambda = \frac{\hbar}{4m}\left(\frac{\beta}{\alpha}\right)^{1/2}. \tag{22}$$

Comparing (19) with the ordinary Schrödinger equation, we find for α the expression

$$\alpha = \frac{2m}{\hbar^2} E . \tag{23}$$

One may distinguish the following cases:

a) $E < 0$ (closed trajectories) corresponds to spontaneous symmetry breaking. In this case, the gravitational Meissner effect for a test particle corresponds to its bounded states;

b) $E > 0$ (open trajectories) corresponds to the absence of spontaneous symmetry breaking. In this case, the "space crystallization" for a test particle corresponds to its free state. If $\alpha = k_x^2$, with k_x the wave number, and $E = p^2/2m$, then from (23) $p = \hbar k_x$ results. Therefore, the wave-particle duality is a property of space-time.

Every system of particles which are moving on geodesic lines can be named, in this way, *gravitational superconducting*, since there are no gravitational forces or gravitational external forces acting on the system. Such a system is the dust field universe. In the case in which the particles are moving on open trajectories, there is no gravitational Meissner effect; it exists only for closed particles trajectories. The transition from one case to another can be realized only through spontaneous symmetry breaking. Both cases correspond to the gravitational superconductor.

3. – The gravitational fluxon

Let us suppose for the system (18)-(19) the solution as

$$\mathbf{A}_{\mathrm{g}} = A(r)\,\mathbf{u}\,, \qquad \Psi = R(r)\, e^{in\theta} \tag{24}$$

where

$$r^2 = x^2 + y^2 \tag{25}$$

and θ is the rotation angle (phase), $\mathbf{u}$ is the unit vector normal to the radius vector and n is an integer. In these conditions eqs. (18) and (19) become

$$-\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(rA)\right] + \left(\frac{16m^2}{\hbar^2} + \frac{4mn}{r\hbar}\right)R^2 = 0\,, \tag{26}$$

$$-\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}R}{\mathrm{d}r}\right) + \left[\left(\frac{n}{r} - \frac{4m}{\hbar}A\right)^2 + \beta\left(R^2 - \frac{\alpha}{\beta}\right)\right]R = 0\,. \tag{27}$$

The most general solutions for the system (26)-(27) are soliton solutions (vortices). A particular solution for this system is

$$A = \frac{n\hbar}{4mr}\,, \qquad \Psi = \sqrt{\frac{-\alpha}{\beta}}\, e^{in\theta}. \tag{28}$$

From this, there follows:

$$\Phi_{\mathrm{g}} = \iint_{R^2} \mathbf{B}\cdot \mathrm{d}\mathbf{S} = \int_{S^1 = \partial R^2} \mathbf{A}\cdot \mathrm{d}l = \frac{n\hbar}{4m}\int_0^{2\pi} \mathrm{d}\theta = n\frac{\pi\hbar}{2m}\,, \tag{29}$$

where R^2 is a region bounded by S^1, l the circumference of the circle of radius R. (29) defines the quantum gravitomagnetic flux

$$\Phi_{\mathrm{g}} = n\Phi_{\mathrm{g}0}, \tag{30}$$

where

$$\Phi_{\mathrm{g}} = \frac{\pi\hbar}{2m} = \iint_{S_0} \mathbf{B}\cdot \mathrm{d}\mathbf{S} \tag{31}$$

is the gravitational fluxoid, $S_0 = 4\pi R_0^2$ is the elementary fluxoid surface and n is an integer identified with the number of flux quanta. Some of the implications of the fluxoid were studied in [9].

4. – Motion of a microparticle in a homogeneous gravitomagnetic field

Let us consider a homogeneous gravitomagnetic field oriented along the Ox axes, expressed as $A_{\mathrm{g}} = B_{\mathrm{g}} x$. Then the Schrödinger equation for a mass microparticle m, situated in this field, takes the form

$$-\frac{\hbar}{2m}\left[\frac{\partial^2 \Psi}{\partial x^2} + \left(\frac{\partial}{\partial y} + \frac{4im}{\hbar} B_{\mathrm{g}} x\right)^2 \Psi + \frac{\partial^2 \Psi}{\partial z^2}\right] = \varepsilon\Psi\,, \tag{32}$$

where ε are the eigenvalues of energy.

The solution of this equation will take the form

$$\Psi(x, y, z) = \varphi(x)\, e^{i(k_y y + k_z z)}. \tag{33}$$

By substituting (30) in (29), there results

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\varphi}{\mathrm{d}x^2} + \frac{1}{2} m\omega_{\mathrm{g}}^2 (x - x_0)^2 \varphi = \varepsilon_l \varphi\,, \tag{34}$$

where

$$\omega_{\mathrm{g}} = 4B_{\mathrm{g}}, \qquad x_0 = -\frac{\hbar}{4mB_{\mathrm{g}}} k_y, \qquad \varepsilon_l = \varepsilon - \frac{\hbar^2 k_z^2}{2m}\,. \tag{35}$$

Equation (31) coincides with Schrödinger's equation for the linear mass oscillator m and pulsation ω_g. In this case, the eigenvalues of energy will be

$$\varepsilon_1 = \left(n + \frac{1}{2}\right)\hbar\omega_g \tag{36}$$

while the eigenvalue corresponding function will be expressed as

$$\varphi(x) = \frac{e^{-(1/2)((x-x_0)/\lambda)^2}}{\sqrt{\lambda}} H_n\left(\frac{x-x_0}{\lambda}\right), \tag{37}$$

where $\lambda = \sqrt{\hbar/m\omega_g}$, and H_n are Hermite polynomials of order n. From (36), it results that:

– the energy of a microparticle found in a gravitomagnetic field is quantified according to the relation

$$\varepsilon = \left(n + \frac{1}{2}\right)\hbar\omega_g + \frac{\hbar k_z^2}{2m}; \tag{38}$$

– for $n = 0$ and $k_z = 0$, relation (35) evidences a certain shifting of the energetic spectrum

$$\varepsilon = \frac{1}{2}\hbar\omega_g. \tag{39}$$

For the Earth gravitomagnetic field, since $B_g \cong 10^{-14}$ [10] this shifting has an experimental undetectable value which is of magnitude $\varepsilon \cong 10^{-48}$ J.

5. – The quantum gravitational Hall effect

The results presented in sect. 4 indicate the fact that, in a superfluid found in a homogeneous gravitomagnetic field, a gravitational, Landau-type dispersion may possibly appear. In a certain case, analogously with the electromagnetic field [11], a "Hall resistance" of a gravitational type may be defined with the relation

$$R_{Hg} = \frac{\Phi_g}{Nm}, \tag{40}$$

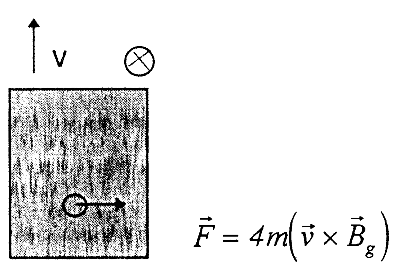

Fig. 1.

where $N = np$ represents the number of particles, n is the number of states corresponding to a Landau level, and p is the number of particles associated to a flow line. Then on considering (29), (40) becomes

$$R_{\mathrm{Hg}} = n \frac{\hbar}{4m} \frac{1}{nmp} = \frac{\hbar}{4m^2 p}, \tag{41}$$

where p is an integer. Such an experiment may be developed through shifting a superfluid within a homogeneous gravitomagnetic field (fig. 1).

Theoretically, to detect such a phenomenon the condition $kT_{\mathrm{c}} < (1/2)\, h\omega_{\mathrm{g}}$ must be satisfied, where k is Boltzmann's constant. For the Earth field, this implies $T_{\mathrm{c}} < 10^{-25}$ K, impossible to achieve in terrestrial laboratories. However, if considering [12], such a phenomenon may be observable at cosmic level.

6. – Conclusion

The linear approximation of the gravitational field permitted the study of the following aspects:

– the connection between the spontaneous symmetry breaking and the absence of the gravitational Meissner effect;

– the gravitational fluxon;

– the motion of a microparticle within a homogeneous gravitational field;

– the quantum gravitational Hall effect.

REFERENCES

[1] PENG H., *Gen. Relativ. Gravit.*, **15** (1983) 725; **22** (1990) 609.
[2] CIUBOTARIU C., *Phys. Lett. A*, **158** (1991) 27.
[3] CIUBOTARIU C., GOTTLIEB I., GRIGA V., CIUBOTARIU C. I. and CROITORU S., *Tensor, NS*, **52** (1993) 138.
[4] CIUBOTARIU C. and AGOP M., *Gen. Relativ. Gravit.*, **28** (1996) 1.
[5] LANDAU L. D. and LIFSIT E. M., *Mecanica* (Ed. Tehnica, Bucuresti) 1996.
[6] NELSON E., *Phys. Rev.*, **150** (1996) 1079.
[7] HO V. B. and MORGAN M. J., *Austr. J. Phys.*, **42** (1992) 245.
[8] FELSAGER B., *Geometry, Particles and Fields* (Odesse University Press) 1981.
[9] AGOP M., CIUBOTARIU C., BUZEA C., BUZEA C. GH. and CIOBANU B., *Austr. J. Phys.*, **48** (1996) 3.
[10] LJUBCIC A. and LOGAN B. A., *Phys. Lett. A*, **172** (1992) 3.
[11] BURNS G., *High-Temperature Superconductivity. An Introduction* (Academie, San Diego) 1992.
[12] LINET D., *Phys. Lett. A*, **146** (1990) 159.