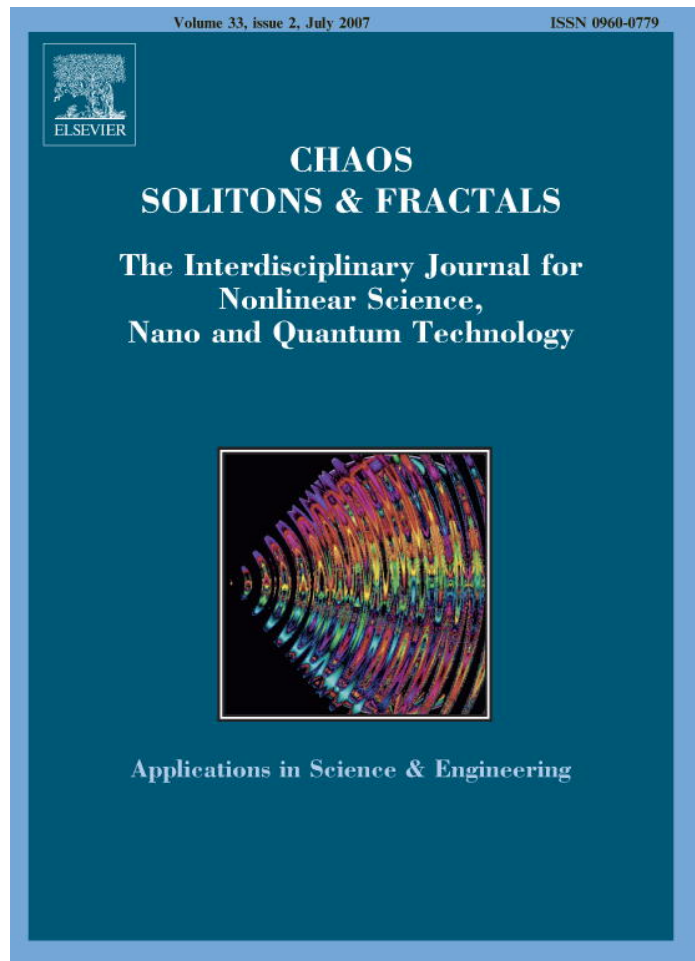


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Chaos, Solitons and Fractals 33 (2007) 396–404

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Chaotic features in Romanian transition economy as reflected onto the currency exchange rate

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Accepted 29 December 2005

Abstract

This work is focused on the study of the existence of elements of deterministic chaos in the exchange rate of Romanian national currency (ROL) with respect to the United States dollar (USD). The temporal evolution between 1 January 1990 and 30 June 2005 is related to the particular Romanian economic transition from a centralized economy toward an open system. While insulating the short run behaviour, we consider the correlation dimension, the positive largest Lyapunov exponent and the Hurst exponent as the most important pointers for chaotic dynamics. By taking into account the main events occurring in the political and economical environment, we split the nearly 16 years period in two intervals, that we classify as “passive transition” and “active transition”. Despite of several quantitative differences, we find evidence of chaotic dynamics in both of them. We also find arguments to state that Romania is close to reaching a functional market economy.

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1. Introduction

The emerging field of econo-physics [1,2] mainly consists of autonomous physico-mathematical models that apply to the financial markets. They are now currently used to explain particular aspects of the complex nonlinear dynamics of stock markets [3], interest rates, money supplies [4], and price levels, as well as the exchange rates [5,6] dynamics. Existence of chaotic behaviour in economics [7] has strong implications for the predictive ability of the time series [8] short time deviation may be important for financial gamblers, while the long term trend can be critical to strategic investors. If the purpose of economic theory is either to develop models that can explain observed regularities or to forecast long term tendencies, the time analysis of the financial series was introduced especially for short run predictions; sometimes it might reveal prerequisites for unexpected phenomena that cause shocks in hot financial systems [9]. The shocks and crisis are frequent phenomena in economic systems subjected to transition. In the last 20 years the countries of central and eastern Europe confronted with such challenges and offered historical examples related to a new type of economic evolution, i.e. from economy of command to free market economy. Differing from the majority of the countries which mostly followed step-by-step processes, Romania suffered a complete and abrupt break, so its evolution has been

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focused to recovering the macroeconomic equilibriums as fast as possible. Due to its ubiquitous existence over time, a practical way to study what it happened is to examine the exchange rate with respect to an adequate reference. Thus, we consider that the exchange rate ROL/USD is a convenient data set for time series analysis, especially as regards the deterministic chaos [10,11].

Chaotic systems are associated with nonlinear deterministic processes exhibiting strong dependence on initial conditions and, under stationary assumptions, are orbiting in the vicinity of some low-dimensional attractors. Hence, one way of revealing the existence of deterministic chaos in a time series is to measure the degree of divergence of nearby orbits in phase-space and the correlation dimension. Such divergence can be measured by the Lyapunov exponents [12], and the presence of at least one positive exponent is a necessary condition of chaos [13]. Thus, in principle, the largest Lyapunov exponent should allow regular and chaotic behaviours to be distinguished.

It has been proved the possibility to extract information about a complex dynamical system, which generates several observed time series, by using only one of them. The idea is to analyze not the given dynamic system, which remains mostly unknown, but an image-system with the same topology that preserves the main characteristics of the genuine one [14]. As it is stated in the literature, there are many attempts to simulate a minimal model, almost all of them being based on the Ruelle-Takens' embedding theorem [15]. The simulation has to follow two steps: the reconstruction of the phase space where the image-system is evolving, and the evaluation of various chaos identifiers particularly, the largest Lyapunov exponent. In turn, the reconstruction involves at least two aspects: the proper choice of the dimension for the reconstructed phase space, i.e. the embedding dimension, and the evaluation of the correlation dimension referring to the degree of complexity expressed by the minimum number of variables that is needed to replicate the dynamic system [16].

The economic systems in transition were subjected to various analysis [17,18]. Despite a certain skepticism related to the inadequate length and quality of the samples that may lead to an illicit application of the techniques, we assume that a time series exhibiting an irregular pattern, nonlinear dependence, low estimates for correlation dimension and positive estimate of largest Lyapunov exponent points out the chaotic dynamics of the underlying system.

2. Data preparation

2.1. Romanian environment

Romania is one of the eastern European countries passing from the command economy to the competing-based market. On the 1st of July 2005 the Central Bank proceeded to the denomination of national currency "leu" (meaning lion) and the Romanian old Lion ROL became Romanian New lion RON,

$$10000 \text{ ROL} = 1 \text{ RON}. \quad (1)$$

In order to avoid possible misunderstandings, and for a better visibility, the absolute values of USD are expressed in the old symbol ROL.

In Fig. 1 we present the time series of ROL/USD exchange rate covering the interval between January 1990 and 30 June 2005 i.e. 4002 daily samples. To facilitate the tracing both year by year and day by day evolution we draw two time axes. For clarity, we use either linear linear (Fig. 1a) and logarithmic scale (Fig. 1b) for the exchange rate. As usual, "holes" from weekends and holidays are ignored and analysis concentrates on trading days. For the period after the middle of 1997, when the Central Bank has announced the daily average parities of ROL with respect to the most important international currencies, we take these numerical values [19], while for the previous interval we consider an averaged inter-banking exchange rate. However, there are no significant differences between them since both of them are reflecting daily averages.

By simple visual inspection of the evolution of the exchange rate (see Fig. 1), and keeping the year as the partition unit, one can a priori identify at least two intervals: the first, with time ranging between 1990 and 2001 (12 years), with a constant, positive trend and the second, with time ranging between 2002 and 2005 (at least 4 years), when a certain stabilization occurs. However, we have to inspect more profoundly the history of the facts.

After the social and political events that provoked the fall of the totalitarian regime (December 1989), the first interval is characterized by huge jumps due to the collapse of production. The inertia of the total deterministic evolution of the centralized economy that has been legally operated until the end of 1989 prolonged its effect for almost two more years, when the working of the economy as a "national holding" diminished. On the other side, the price liberalization rapidly began to manifest its effects. Specific features of transition are the "three digits inflation" (256.1% in 1993) and the fast depreciation of the national currency (from 14 ROL/USD in December 1989 up to 33 540 ROL/USD in 2003). Thus, a convenient pointer for the variation of the exchange rate is the relative variation $\Delta_j(i)$ given by

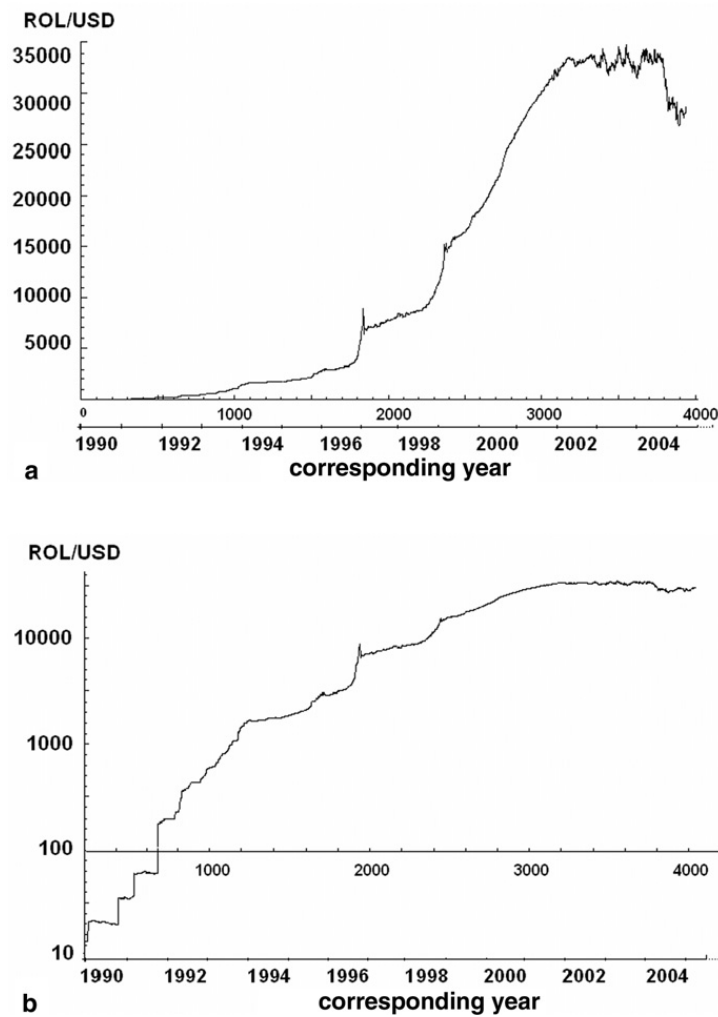


Fig. 1. Absolute values ROL/USD exchange rates: linear scale (a) and logarithmic scale (b).

$$\Delta_y(i) = \frac{y(i) - y(i-1)}{y(i-1)} \quad \text{for } i = 1, \dots, N, \quad (2)$$

where $y(i)$ is the ROL/USD exchange rate in the i th day, and N is the total number of measuring days.

There are four huge jumps (two in 1990 and two in 1991, see Fig. 2) reflecting the accumulation of the demand pressure upon the slow-developing (in fact no developing) supply that led to an increased import of goods; the insufficient feed-forward of a weak monetary system couldn't balance the long response time of production, and explosive corrections occurred. These four jumps are the greatest among the whole 15 years period under investigation.

The fusion of the exchange rates by the end of 1991 institutionalized the price liberalization and enforced ROL to reach a limited convertibility; the cost was a strong devaluation. For these reasons we keep caution and, eventually, we neglect the entire interval 1 January 1990–31 December 1991.

In the following 5 years, the pressure of the market forces continues, and the necessity for structural changes became stringent. Limited laws of property and timid attempts to transfer the state property to private ownership were too little regulations against the imperative necessity of an adequate infrastructure capable to handle the competing environment. The quasi-parabolic shaped dependence of the curve (Fig. 1a) shows a much more responsive behaviour due to the increasing power of the Central Bank, but there still exist several jumps. The most important took place on February 1997, after the new elected political regime elaborated a coherent policy in order to support the structural changes. The Central Bank diminished its interventionist policy and began to use open market instruments, and ROL became fully convertible for current account transactions (the beginning of the year 1998). Thus, we consider the end of 1997 as an apparent border between two distinct periods: the first, between 1990 and 1997 is the “passive

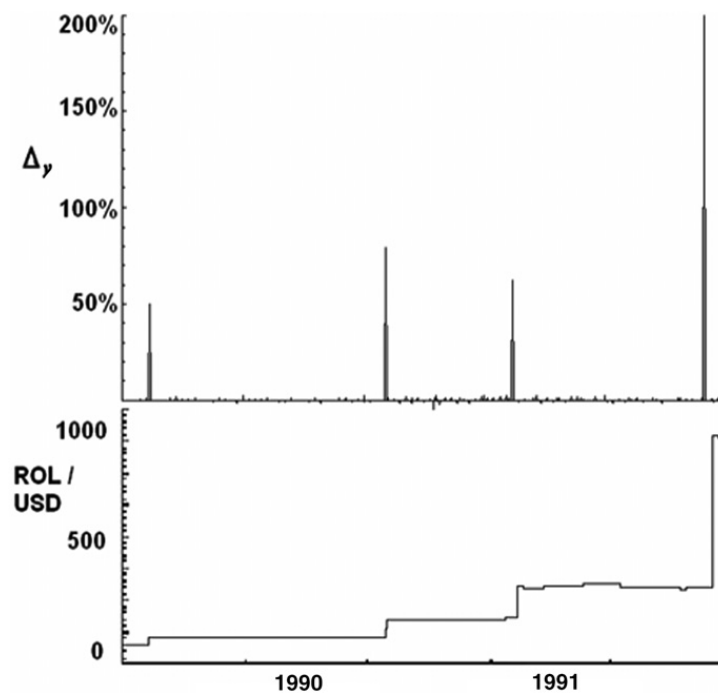


Fig. 2. Four jumps of ROL/USD exchange rates: relative variation (upper graph) and the corresponding absolute values (lower graph).

transition” with little and limited remedial measures, and the second, beginning with 1998 is the “active transition”, with clear objectives and more rigorous forecasting.

The second half of the year 2000 might be considered the end of the implementations of the structural changes. One can observe a qualitative change in the concavity of the shape: while new macro-economical equilibriums were settling down, they modify the underlying dynamics, and the curve moves away from its characteristic long run, quasi parabolic appearance.

The steady-state regime seems to begin with the year 2001. An important event took place in March 2005, when the Central Bank announced an almost nil interventionist policy with respect to the exchange rate in order to obtain the total convertibility of ROL, i.e. including the capital account transactions.

So far we have mentioned arguments to split the transition into more distinct periods. However, the objectives of the following analysis are: the first, to find out if there is any correspondence between the historical events and the time evolution of the exchange rate, and the second, if any nonlinear dynamics is observed, to try to characterize the main stages of the transition from the point of view of the deterministic chaos.

2.2. Data detrending

Unlike laboratory experiments, in natural sciences there is no way to maintain steady flows in economic growth and to describe business cycles by invariant attractors. Moreover, a transition époque is essentially a nonstationary one, and macroeconomic time series are not reproducible in history. The structural changes such as technological transfers or innovations, the implementations of new monetary and financial instruments, changes in the operating procedure of the Central Bank did influence the Romanian economy in a quite complicated and, very probable, nonlinear manner. A proper decomposition of trend and cycles may find an appropriate scheme to weigh the short- and long run impacts of economic movements on economic dynamics. In order to isolate the long run and the short run behaviours, an adequate detrend is necessary. Assuming that the economic science is considering “long run” to be “greater than” 1 year, the detrending criterion appears naturally: the baseline function $T(i)$ has to be responsive enough to the fast varying slopes and tails but with cycles no shorter than 250 sampling points i.e. 1 year. A thirteen degree polynomial fits the long-run trend over the whole domain (Fig. 3). In order to avoid the oscillatory approximation that induces artificial variation for small values of i , an adequate prolongation was performed; the exchange rate residuals are

$$x(i) = y(i) - T(i) \quad \text{for } i = 1, \dots, N. \quad (3)$$

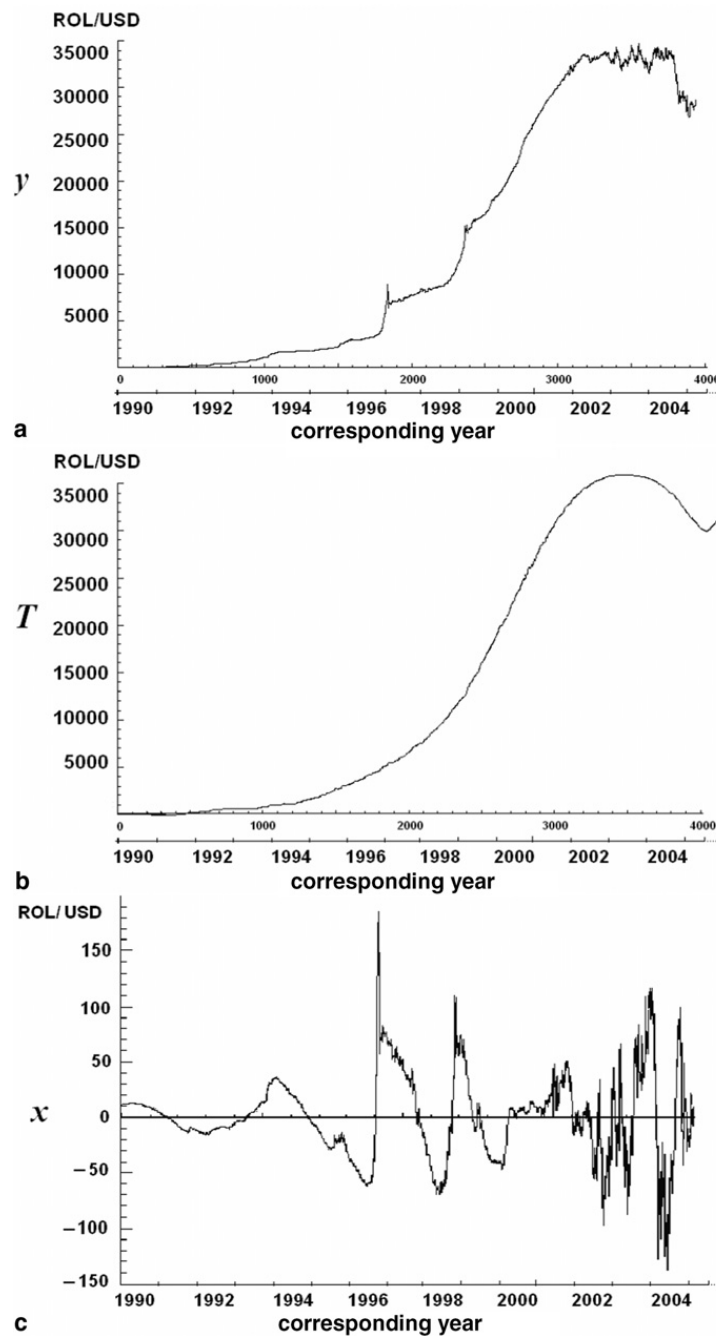


Fig. 3. Exchange rate $y(i)$ (a), trend function $T(i)$ (b), and detrended exchange rate $x(i) = y(i) - T(i)$ (c).

2.3. Procedure

It is known [20] that only nonlinear, deterministic signals are characterized by low-dimensional attractors, finite, positive Lyapunov exponents and good short-term nonlinear predictability. We analyze the chaotic behaviour by performing three main tasks: (i) the test for causality via the Brock–Dechert–Scheinkman (BDS) test [21] in order to distinguish the purely random process from a deterministic one; (ii) the estimation of the correlation dimension D_{corr} of the attractor; (iii) the estimation of the largest Lyapunov exponent λ_{max} . Additionally, as the fourth task, we compute the Hurst exponent not for evidence of scaling properties but for checking the persistence of the series.

The BDS test is checking the null hypothesis of whiteness (independent, identically distributed samples) against an unspecified alternative. It is effective for proving nonlinear determinism only if other three features: nonstationarity, linear dependence, and stochastic behaviour were rejected. We assume that the detrending procedure completely filters

out the first and the second features; for the third, supplemental investigations are necessary. However, this is still an open subject in the literature. Briefly, we only limit ourselves to state if the null hypothesis is admitted or rejected. We mention that the values for the BDS test are negative for purely random samples.

The correlation dimension that gives the dimension of the attractor is

$$D_{\text{corr}} = \lim_{\varepsilon \rightarrow 0} \left| \frac{\log C(N, m, \varepsilon)}{\log \varepsilon} \right|, \tag{4}$$

where $C(N, m, \varepsilon)$ is the correlation function of the m -dimensional reconstructed vectors in the embedding space [22] and ε is the radius of the infinitesimal hypersphere.

The largest Lyapunov exponent which indicates the time evolution of the distance $d[X(i_0) \rightarrow X(i_0 + i)]$ between the m -dimensional, reconstructed vectors X , after i steps, as indicated in the Rosenstein algorithm [23] is

$$\max_{i_0} \{d[X(i_0) \rightarrow X(i_0 + i)]\} \sim i^{i^{\text{max}}}. \tag{5}$$

The Hurst exponent H , defined by the time evolution of the root-mean-square fluctuation of the averaged displacement over all possible positions i_0 [24] (the bars indicate the average) is

$$\{[\overline{[x(i_0 + i) - x(i)]^2} - \overline{[x(i_0 + i) - x(i)]^2}]^{\frac{1}{2}}\} \sim i^H. \tag{6}$$

A four-year running window analysis was performed over the entire interval January 1990–June 2005 in order to identify significant changes in the values of the above mentioned quantities. The temporal length is imposed by the requirement to have at least 1000 samples for every computation in order to obtain significant conclusions.

3. Results

The four-year window has been moved over the entire time interval with a step of 1 year; the numerical values are summarized in Table 1.

By plotting the slope of the linear part of the curves $(\log \varepsilon, C(N, m, \varepsilon))$ in a (m, D_{corr}) diagram, the saturation occurs in almost all cases, so the correlation dimension is well defined, indicating the presence of an important deterministic component [4]. The exception is the window no.1, ranging between 1990 and 1993, where the plateau seems to maintain a small positive slope. This might be the consequence of a stochastic component. We stress that the lack of an adequate institutional feed-forward in the economy and the social pressure led to political decision to maintain the exchange rate at artificial constant values for long time periods (Fig. 2 lower graph). This is a supplemental argument for dropping out the years 1990 and 1991 in the final partition.

The embedding dimension and, consequently, the correlation dimension increase with time along the entire range. The BDS test value is taken for the correspondent embedding dimension and is consistent with the rejection of the null hypothesis in all cases.

Table 1
Numerical values obtained with a four years window sampling

Window no.	Time range	Embedding dimension m	BDS test value	Correlation dimension, D_{corr}	Lyapunov exponent, λ_{max}	Hurst exponent, H	Relative variation ^a , Δ_y (%)
1	1990–1993	4	0.329	1.68	0.139 ± 0.039	0.528	9014.30
2	1991–1994	4	0.252	1.73	0.121 ± 0.035	0.609	4867.70
3	1992–1995	5	0.259	1.99	0.138 ± 0.031	0.640	1256.80
4	1993–1996	5	0.234	1.71	0.113 ± 0.032	0.569	802.17
5	1994–1997	5	0.154	1.80	0.046 ± 0.031	0.657	523.09
6	1995–1998	5	0.386	2.24	0.079 ± 0.032	0.653	517.77
7	1996–1999	5	0.241	2.20	0.111 ± 0.034	0.635	586.28
8	1997–2000	6	0.275	2.53	0.117 ± 0.031	0.611	524.72
9	1998–2001	7	0.133	3.09	0.108 ± 0.022	0.614	289.08
10	1999–2002	7	0.120	3.16	0.113 ± 0.025	0.550	202.46
11	2000–2003	7	0.113	3.42	0.123 ± 0.023	0.481	78.36
12	2001–2004	7	0.124	3.20	0.116 ± 0.026	0.489	11.55
13	2002–2005	9	0.146	4.45	0.118 ± 0.022	0.477	–5.89

^a The relative variation is calculated over the window period, i.e. 4 years.

Positive largest Lyapunov exponents are found everywhere but the value in the window no. 5 it is somehow uncertain because of the magnitude of the error.

The quasi-oscillatory, short run features that begin with the year 2000 and enhances in 2002 are provoking the decrease of the global Hurst exponent. It is interesting to remark that the short run anti-persistence, pointed out by a value of the Hurst exponent smaller than 0.5, seems to be consistent with the long-run tendency reflected by the function $T(t)$: the baseline changes its concavity approximately at the same time when the Hurst exponent passes over the threshold value of 0.5.

A cross analysis of the preceding quantities reveals indeed two characteristic intervals of the transition that approximately match the qualitative considerations we have made upon the Romanian environment. To be more specific, the “passive transition” and the “active transition” get support and consistency over other scenarios. The interval no. I, 1990–1997 is characterized by low-correlation dimension i.e. simple underlying dynamics, weak sensitivity to initial conditions, positive long-run trend. We include the year 1997 in this interval due to three reasons: the first, its strong influence upon the uncertainty of the Lyapunov exponent both in the window no. 5 and in the entire interval; the second, it is the last year with a “three digit” inflation index (after that, the four-year relative variation exhibits a significant fall—see Table 1); and the third, the delay between the implementation of support policies for structural changes and their perceptible effects bring it closer to the hesitant and contradictory époque (“passive transition”).

Because we are right in suspecting the first 2 years, 1990 and 1991, of some linear stochastic kind of dependence, we are reticent to consider the whole interval as a chaotic one. If we reduce the analysis to the remaining 6 years, i.e. 1992–1997, the main characteristics remain almost the same, but we have stronger arguments for nonlinear dependence. Despite the small increase of the value of the Lyapunov exponent being balanced by the increase of its uncertainty, we can state that the exchange rate is showing chaotic dynamics with weak sensitivity to initial conditions. Thus, the exchange rate is predictable not only from the linear point of view (expressed by the linear term of function $T(t)$), but also from the nonlinear dynamics perspective. The results are summarized in Table 2.

Interval no. II, 1998 to June 2005 is characterized by increased correlation dimension, i.e. more complex dynamics, and clear positive values for the largest Lyapunov exponent. All the usual hypotheses for chaotic dynamics are fulfilled, and the exchange rate is less predictable than in the previous case. The value of the Hurst exponent smaller, but close to 0.5 confirms a higher volatility of the series. This is somehow astonishing because, at least for the first part of the inter-

Table 2
Numerical values for the two characteristic intervals (see text)

Interval no.	Time range	BDS test value	Embedding dimension, m	Correlation dimension, D_{corr}	Lyapunov exponent, λ_{max}	Hurst exponent, H
I	1990–1997	0.259	4	1.44	0.045 ± 0.021	0.643
I reduced	1992–1997	0.190	4	1.55	0.054 ± 0.032	0.648
II	1998 to 30 June 2005	0.296	6	2.76	0.170 ± 0.025	0.488

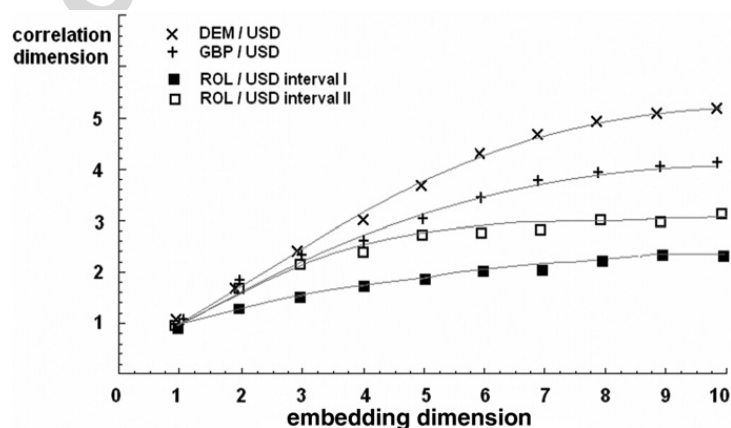


Fig. 4. Correlation dimension for ROL–USD, GBP–USD and DEM–USD exchange rates.

val no. II, to be more specific for the range 1998–2002, the positive long-run trend continues. Despite being about a hundred times smaller than for the interval no. I, the linear prediction is indicating an averaged relative variation per year which remained large (72.27% compared to 7264.29%).

Interval no. II is the domain with significant functional changes toward the open market economy. The transition seems to come to end during 2005, even if the underlying structure is not fully developed. As stated in literature [25], a free market economy in a steady state regime may be analyzed via a reconstructed space having up to 10–12 dimensions which contains a strange attractor with a correlation dimension around 4.5. In Fig. 4 are represented the ROL/USD exchange rates for the partitioned intervals, as well as the exchange rates for other two European countries with the same USD reference [5]: German mark (DEM for the interval 1973–1998) and Great Britain pound (GBP for the interval 1973–2000). Both of them are exhibiting higher correlation dimensions, corresponding to more complex dynamics.

4. Conclusions

The study covers almost 16 years of transition from the command economy to a functional market economy of an East-European country. The measurable effects are registered via the exchange rate ROL/USD read-out function. The detrending procedure isolates the short- from the long-run behaviour. Provided that the time series is not an independent, identically distributed collection of samples, i.e. the BDS test rejects the null hypotheses, the analysis is focused upon two more parameters which characterize the chaotic evolution: the correlation dimension and the largest Lyapunov exponent. Additionally, the Hurst exponent reveals the persistence of the series.

While identifying strong indication of determinism over the whole time range, our study is revealing quite sharp distinctions along the time evolution and allows us to define two time intervals that reflect the main characteristics of the developing underlying dynamics. This partition is consistent with the historical chronology of the most important political and economical events.

The residuals of the command economy and the hesitant macroeconomic policies imprinted a stepped growing trend for the exchange rate for almost 2 years (1990–1991). In the following 6 years (1992–1997) the market demand are pressing the monetary system and the characteristics of the deterministic chaos and short-run predictability became apparent in the foreign exchange currency. This was the interval classified as “passive transition”.

The last interval 1998 to 30 June 2005 (classified as “active transition”) reflects the coherent foresight policies and the implementation of the structural changes with major functional consequences; the last year might represent the beginning of a quasi-stationary regime. The economic infrastructure and the monetary instruments are growing in complexity and number, tending to get the main features of the functional market systems. Neglecting the long-run behaviour, the residuals of the exchange rate have the main features of deterministic chaos: nonlinear dependence, low-correlation dimension and positive values for the largest Lyapunov exponent.

The results of our study provide arguments to state that Romania is close to reaching a functional market economy.

Acknowledgement

The authors are grateful to Prof. Gheorghe Oprescu for his valuable observations and suggestions. This work was supported by the Grant CNCSIS 287/2005.

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