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Problems:			

GENERAL PHYSICS COMPETITION FOR ENGINEERING STUDENTS "ION I. AGARBICEANU"

XI Edition 2023 13 May 2023

Theoretical test, Physical Section 2

Each contestant participates in the contest with 3 of the 6 subjects of their choice. On the first competition sheet, the candidate will specify under his signature the numbers of the subjects he has chosen.

1. Consider a particle of mass m located in a one-dimensional potential pit with infinite walls. Inside the pit $V=0$ for $|x|<L$. The wave function of the particle in the potential well is. $\psi(x)=A\sin kx+B\cos kx$.

a) Apply the boundary conditions to the wave function and show that $k=\dfrac{n\pi}{2L}$, $n=1,2,3,\ldots$, cu $A=0$ for odd values of *n* impar and $B=0$ for even values of *n*.

b) The particle localization probability density at a point x in space is equal to .. $|\psi(x)|^2$. Use this to normalize the wave function, i.e. determine the normalization constants A or B

c) Use the result obtained in c) to calculate the mean value and uncertainty of *x* as a function of *n*. Show that for very large values of *n* the results tend to the classical values obtained for a particle moving back and forth in a pit, with constant speed $\langle x\rangle=0$, $\langle x^2\rangle^{1/2}=L/\sqrt{3}$.

d) The Schrödinger equation can be written as $\left(\frac{\hat{p}^2}{2m}+V\right)\psi = E\psi$, where $\hat{p} = -i\hbar\frac{\partial}{\partial x}$ for unidimensional case. Show that in n state the particle's energy is $E_n = \frac{\pi^2\hbar^2}{8mL^2}n^2$.

e) The first transition in the Lyman series of hydrogen has a wavelength equal to 121.6 nm. Using the one-dimensional potential well model, estimate a characteristic size of the hydrogen atom.

PO 2 p.

a) Bundary conditions are $\psi(L) = \psi(-L) = 0$, so we get

$B\cos kL + A\sin kL = 0$ And 1 p.

$B\cos kL - A\sin kL = 0$ 1 p.

Adding and subtracting the previous relations we obtained $B=0$ and $\sin kL = 0$ or $A=0$ and $\cos kL = 0$, so $k = \frac{n\pi}{2L}$, $n = 1, 2, 3, \ldots$, with $A=0$ for odd n and $B=0$ for even n. 3 p.

It is observed that the wave functions have a well-specified parity, i.e. they are odd for n even and even for n odd.

b)We must state that $\int_{-L}^{L}|A|^2\sin^2 kx\,dx = 1$, for odd n or $\int_{-L}^{L}|B|^2\cos^2 kx\,dx = 1$, for even n. Since $\cos^2 y = \frac{1+\cos 2y}{2}$ and $\sin^2 y = \frac{1-\cos 2y}{2}$, we get $|A| = |B| = \frac{1}{\sqrt{L}}$, phases of A and B being arbitrary and insignificant. 2 p.

c) Due to the symmetry of the problem (potential pit symmetrical to the origin) it follows that $\langle x\rangle = 0$. The standard deviation of the position is $\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$, se we only need to calculate $\langle x^2\rangle$

$$\langle x^2\rangle = \frac{1}{L}\int_{-L}^{L} x^2 \sin^2\left(\frac{n\pi x}{2L}\right)dx,$$ 1 p.

for n even and similarly for n odd, changing the sine function to the cosine function. Integrating by parts, we get

$$\langle x^2\rangle = \frac{L^2}{3}\left(1-\frac{6}{n^2\pi^2}\right)$$ 2 p.

for every value of n.

Classic problem gives $\langle x^2\rangle = \frac{1}{2L}\int_{-L}^{L} x^2 dx = \frac{L^2}{3}$.

d) Inside the well we have $V = 0$, so we get $E = \frac{\hbar^2 k^2}{2m}$, for a known k in the specific boundary conditions.

2 p.

e) Considering the energy difference of the states with $n = 1$ and $n = 2$, as the energy of the first transition in Lyman series we get

$$E_2 - E_1 = \frac{3\pi^2\hbar^2}{8mL^2} = \frac{2\pi\hbar c}{\lambda}.$$ 4 p.

The effective dimension of the hydrogen atom is>

$$L = \sqrt{\frac{3\pi\hbar\lambda}{16mc}} \cong 1{,}59\times 10^{-10}\,\text{m}.$$ 2 p.

2. An electron microscope can separately distinguish two points located at a d distance. If this distance satisfies the condition $d \geq \frac{\lambda}{2A}$, where λ is the de Broglie wavelength of the electrons, and A is a constant of the apparatus, called the numerical aperture. Knowing the electron acceleration voltage $U = 100\,\text{kV}$ and $A = 0{,}15$, determine the minimum value of d considering a relativistic electron movement.. Se cunosc: $m_{oe} = 9{,}1 \times 10^{-31}$ kg , $c \cong 3\cdot 10^8$ m/s şi .$h = 6{,}626 \times 10^{-34}$Js

PO 2 p.

The relativistic momentum of the electron is>

$$p = \frac{m_{0e}v}{\sqrt{1-\frac{v^2}{c^2}}},$$ (1) 2 p.

so the deBroglie wavelength is

$$\lambda = \frac{h}{p} = \frac{h\sqrt{1-\frac{v^2}{c^2}}}{m_{0e}v}.$$ (2) 2 p.

The kinetic energy of the electron is

$$\frac{m_{0e}c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_{0e}c^2 = eU,$$ (3) 2 p.

Resulting the velocity>:

$$v = c\frac{\sqrt{\frac{eU}{m_{0e}c^2}\left(2+\frac{eU}{m_{0e}c^2}\right)}}{1+\frac{eU}{m_{0e}c^2}} \quad (4) \qquad 2\text{ p.}$$

Replacing (4) in (2), it gets:

$$\lambda = \frac{h}{m_{0e}c}\frac{1}{\sqrt{2\frac{eU}{m_{0e}c^2}\left(1+\frac{eU}{2m_{0e}c^2}\right)}}. \quad (5) \qquad 2\text{ p.}$$

Since in our case $\frac{eU}{2m_{0e}c^2} << 1$, we can approximate:

$$\frac{1}{\sqrt{1+\frac{eU}{2m_{0e}c^2}}} \cong \frac{1}{1+\frac{eU}{4m_{0e}c^2}} \approx 1-\frac{eU}{4m_{0e}c^2} \quad (6) \qquad 2\text{ p.}$$

Considering (6the wavelength in (5) become:

$$\lambda \cong \frac{h}{\sqrt{2m_{0e}eU}}\left(1-\frac{eU}{4m_{0e}c^2}\right). \quad (7) \qquad 2\text{ p.}$$

Using λ from (7) in the assumption $d \geq \frac{\lambda}{2A}$, we get:

$$d \geq \frac{h}{2A\sqrt{2m_{0e}eU}}\left(1-\frac{eU}{4m_{0e}c^2}\right). \quad (8) \qquad 2\text{ p.}$$

i.e

$d_{\min} \cong 0{,}123$ Stream.

2 p.

3. Knowing that the surface temperature of the human body is$\theta = 36\ ^0C$,calculate the power of radiation emitted by the body. The human body is assumed to behave as a gray body with the emissivity factor $\varepsilon = 0.6$ and the average surface of the human body is $S = 1.5\ m^2$.. How long does it take for a human body equivalent to a mass of water $m = 75$ kg de apă ($c = 4200$ J/kg · K) placed in a vacuum in outer space (at ~0 K) to reach temperature $T_0 = 273\ K$? It is considered that body temperature is uniform and that there are no metabolic reactions to increase temperature. The Stefan-Boltzmann constant is known $\sigma = 5{,}67 \times 10^{-8}\ Wm^{-2}K^{-4}$.

Solution scale :

- ex officio 1.0 p

- the elementary amount of energy radiated by the human body is:

$dW = \Phi dt = S\varepsilon\sigma T^4 dt$ 1.5 p

-Radiation powero emitted by the body:

$P = \frac{dW}{dt} = \varepsilon\sigma S T^4 = 465\ W$ 1.0 p

- numeric :

P = 465 W 0.5 p

- The energy radiated by the body will lower the temperature a. Q.

dW=-mcdT 1.5 p

-Results:

$$S\varepsilon\sigma T^4 dt = -mcdT$$ 1.5.p

- integration obtains:

$\tau = \int_0^{\tau} dt = -\frac{mc}{S\varepsilon\sigma} \cdot \int_T^{T_0} \frac{dT}{T^4} = \frac{mc}{3\varepsilon\sigma S}\left(\frac{1}{T_0^3} - \frac{1}{T^3}\right) = 8.73$ hours 2.0 p

- num $\tau = \int_0^{\tau} dt = 8.73$ hours 1 p

4. Two balls with initial velocities v_{01} and v_{02} are thrown vertically upwards from the surface of a lens converging with focal length f. Determine:

a. Minimum speed (v_{lim}) for which the balls produce real images in the lens
b. The time interval for which the two balls simultaneously produce real images in the lens knowing that v_{01}=nvlim and $v_{02}=(n+1)v_{lim}$
c. The time interval for which only one of the balls produces a real image

ex officio 1.0 p

The real image condition through the lens. is $y \geq f$ 1.0p

We apply the limit condition and get , hence $f = \frac{v_{lim}^2}{2g} v_{lim} = \sqrt{2gf}$ 2p

The solutions of the equation are . $y_1 = v_{01}t - \frac{gt^2}{2} \geq f y_2 = v_{02}t - \frac{gt^2}{2} \geq f$

For the first ball the times are. $t_1^{(1)} = \sqrt{\frac{2f}{g}}\left(n - \sqrt{n^2-1}\right)$ *and* $t_2^{(1)} = \sqrt{\frac{2f}{g}}\left(n + \sqrt{n^2-1}\right)$

2p

The considered time interval is taken between the roots, i.e.. . $\Delta t_1 = t_2^{(1)} - t_1^{(1)} = 2\sqrt{\frac{2f}{g}(n^2-1)}$

1p

For the second ball, solving the equation gives the times. $t_1^{(2)} = \sqrt{\frac{2f}{g}}\left(n+1-\sqrt{n(n+2)}\right) t_2^{(2)} = \sqrt{\frac{2f}{g}}\left(n+1-\sqrt{n(n+2)}\right)$ 1p

It is noted that$t_2^{(2)} > t_2^{(1)}$ and $t_1^{(2)} < t_1^{(1)}$so the range is .$\Delta t_1 = 2\sqrt{\frac{2f}{g}(n^2-1)}$

1p

From the above considerations, the required time frame is: $\Delta t_2 = \left[t_1^{(2)}, t_1^{(1)}\right] \cup \left[t_2^{(1)}, t_2^{(2)}\right]$

1p

5. Two monochromatic waves of equal frequencies propagate in parallel directions perpendicular to an inhomogeneous plate of thickness L. In the portion traversed by the first wave, the refractive index n1(z)=n0 and the second wave enters an area where the refractive index is $n_2(z) = \left(1 + \left(\frac{z}{L}\right)^2\right)$What is the thickness of the wafer so that, at the exit of the plate, the two waves cancel each other out.

ex officio 1.0 p

We write the accumulated phases of each of the two waves propagating through the areas of the plate.

For the first wave we have $\phi_1 = \frac{2\pi}{\lambda}\int_0^L n_0 dz = \frac{2\pi n_0 L}{\lambda}$, where is the wavelength, and λz is the common direction of propagation through the plate. 3p

For the second wave, the accumulated phase is: ..$\phi_2 = \frac{2\pi}{\lambda}\int_0^L n_2 dz = \frac{2\pi}{\lambda}\int_0^L \left(1 + \frac{z^2}{L^2}\right) dz = \frac{2\pi}{\lambda}\left(L + \frac{L^2}{3}\right)$ 3p

For the waves to cancel out, the phase shift between them must be $\Delta\phi = \pi$, or equivalent, $L + \frac{L^2}{3} - n_0 L = \frac{\lambda}{2}$ 2p

hence $L = \frac{3\lambda}{4-3n_0}$ 1p

6. In an enclosure, we insert N_{01} radioactive nuclei with λ_1 activity . The decay product is in turn radioactive with λ_2 activity. Knowing that the final product is stable, find the decay law for the intermediate product and determine the time for which its activity is maximum.

ex officio 1.0 p

The decay law for the first product is: $-dN_1 = \lambda_1 N_1 dt$, hence $N_1 = N_{01}\exp(-\lambda t)$

1p

For the intermediate product we have: $dN_2 = dN_1 - \lambda_2 N_2 dt$. We divide by dt and get: $\frac{dN_2}{dt} = \frac{dN_1}{dt} - \lambda_2 N_2 = \lambda_1 N_1 - \lambda_2 N_2$. We multiply by $\exp(\lambda_2 t)$ in both terms and have:

$\frac{dN_2}{dt}\exp(\lambda_2 t) + N_2\lambda_2 \exp(\lambda_2 t) = \lambda_1 N_1 \exp(\lambda_2 t)$ 1p

The term left is actually the derivative with respect to time of the function $N_2 \exp(\lambda_2 t)$, from which it follows that:

$\frac{d}{dt}(N_2 \exp(\lambda_2 t)) = \lambda_1 N_1 \exp(\lambda_2 t) = \lambda_1 N_{01} \exp\big((\lambda_2 - \lambda_1)t\big)$ 1p

We separate the variables and get:

$d(N_2 \exp(\lambda_2 t)) = \lambda_1 N_{01} \exp\big((\lambda_2 - \lambda_1)t\big)dt$ 1p

We integrate in both terms and get:

$N_2 \exp(\lambda_2 t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{01}\big(\exp\big((\lambda_2 - \lambda_1)t\big) + C\big)$ 1p

The constant of integration is removed from the initial condition $N_2 = 0$ (since initially there is no intermediate product). We have $C = N_2(0) = -1$:. The expression of the intermediate product is:

$N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{01}(\exp(-\lambda_1 t) - \exp(-\lambda_2 t))$ 1p

The time at which activity is maximum is whenN_2 is maximum. Apply the condition of extreme on the above relationship and we obtain:

$\left(\frac{dN_2}{dt}\right)_{max} = 0$ 1p

From derivation we have:

$\frac{\lambda_1 N_{01}}{\lambda_2 - \lambda_1}(-\lambda_1 \exp(-\lambda_1 \tau) + \lambda_2 \exp(\lambda_2 \tau)) = 0$ 1p

The quantity in parentheses equals zero, and is obtained, after calculations: $\tau = \frac{1}{\lambda_1 - \lambda_2}\ln\left(\frac{\lambda_1}{\lambda_2}\right)$

1p