



NAME AND SURNAME

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Selected issues:

**GENERAL PHYSICS COMPETITION FOR ENGINEERING STUDENTS  
"ION I. AGARBICEANU"**

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Theoretical test, Physical Section 2

*Each contestant participates in the contest with 3 of the 6 subjects of their choice. On the first competition sheet, the candidate will specify under his signature the numbers of the subjects he has chosen.*

1. Consider a particle of mass  $m$  in a one-dimensional potential pit with walls inside the pit  $V = 0$  for  $|x| < L$ . The wave function of the particle in the potential pit is of the form  $\psi(x) = A \sin kx + B \cos kx$ .

a) Apply to the wave function the boundary conditions and show that  $k = \frac{n\pi}{2L}$ ,  $n = 1, 2, 3, \dots$ , with  $A = 0$  for odd  $n$  and  $B = 0$  for  $n$  even are obtained.

b) The probability density of locating the particle at a point  $x$  in space is equal to  $|\psi(x)|^2$ . Use this to norm the wave function, i.e. determine the norming constants  $A$  or  $B$ .

c) Use the result obtained in point c) to calculate the mean value and uncertainty of  $x$  as a function of  $n$ . Show that for very large values of  $n$  the results tend to the classical values obtained for a particle moving back and forth in a pit at constant velocity:  $\langle x \rangle = 0$ ,  $\langle x^2 \rangle^{1/2} = L / \sqrt{3}$ .

d) The Schrödinger equation can be written as  $\left( \frac{\hat{p}^2}{2m} + V \right) \psi = E \psi$ , where  $\hat{p} = -i\hbar \frac{\partial}{\partial x}$  for the one-dimensional case. Show that in the  $n$  state the particle has energy  $E_n = \frac{\pi^2 \hbar^2}{8mL^2} n^2$ .

- e) The first transition in the Lyman series of hydrogen has a wavelength equal to 121.6 nm. Using the one-dimensional potential pit model, estimate a characteristic size of the hydrogen atom.
2. An electron microscope can distinguish two points apart  $d$  separately if this distance satisfies the condition  $d \geq \frac{\lambda}{2A}$ , where  $\lambda$  is the de Broglie wavelength of electrons and  $A$  is a constant of the apparatus, called a numerical aperture. Knowing the acceleration voltage of electrons  $U = 100 \text{ kV}$  and that  $A = 0,15$ , determine the minimum value of  $d$ . The electron is thought to move relativistically. They know each other:  $m_{oe} = 9,1 \times 10^{-31} \text{ kg}$ ,  $c \cong 3 \cdot 10^8 \text{ m/s}$  and  $h = 6,626 \times 10^{-34} \text{ Js}$
  3. Knowing that the surface temperature of the human body is, calculate the strength of radiation emitted by the body. It is assumed that the human body behaves like a gray body with the emissivity factor and that the average surface area of the human body is . How long does it take for a human body equivalent to a mass of water ( $m$ ) placed in a vacuum in outer space (at  $\sim 0 \text{ K}$ ) to reach temperature? It is considered that body temperature is uniform and that there are no metabolic reactions to increase temperature. The Stefan-Boltzmann constant is known  $\sigma = 5,67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$ .
  4. Two balls with initial velocities  $v_{01}$  and  $v_{02}$  are thrown vertically upwards from the surface of a lens converging with focal length  $f$ . Determine:
    - a. Minimum speed ( $v_{lim}$ ) for which the balls produce real images in the lens
    - b. The time interval for which the two balls simultaneously produce real images in the lens knowing that  $v_{01} = n v_{lim}$  and  $v_{02} = (n+1) v_{lim}$
    - c. The time interval for which only one of the balls produces a real image
  5. Two monochromatic waves of equal frequencies propagate in parallel directions perpendicular to an inhomogeneous plate of thickness  $L$ . In the portion traversed by the first wave, the refractive index  $n_1(z) = n_0$  and the second wave enters an area where the refractive index is  $n_2(z) = \left(1 + \left(\frac{z}{L}\right)^2\right)$ . What is the thickness of the wafer so that, at the exit of the plate, the two waves cancel each other out.
  6. In an enclosure, we insert  $N_{01}$  radioactive nuclei with  $\lambda_1$  activity. The decay product is in turn radioactive with  $\lambda_2$  activity. Knowing that the final product is stable, find the decay law for the intermediate product and determine the time for which its activity is maximum.